

Dynamics of Rate-of-Return Regulation

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Under rate-of-return regulation, a firm's product prices are constrained by the requirement that investors not earn more than an allowable return on the firm's assets. This paper examines the dynamic properties of the rate-of-return regulation process when the regulated firm periodically undertakes new capacity investments. Our analysis identifies prices that correspond to stationary values of the regulation process. It is shown that the underlying depreciation rules for property, plant, and equipment determine whether these stationary prices will be above, equal to, or below the long-run marginal cost of providing the regulated service. We provide conditions under which the rate-of-return regulation process is dynamically stable so that prices indeed converge to their stationary values. The overall efficiency of this regulation method is shown to depend on how well the applicable depreciation schedule matches the productivity pattern of the assets in use.

Key words: accounting; marginal cost; rate-of-return regulation; depreciation

History: Received December 19, 2009; accepted August 16, 2011, by Gérard P. Cachon, accounting. Published online in *Articles in Advance*.

1. Introduction

Regulatory agencies commonly rely on rate-of-return (RoR) regulation to determine product prices in a range of industries such as telecommunication services, electricity, gas, and water.¹ In its traditional form, RoR regulation sets prices so that the regulated firm's return on assets attains a target value in each period. Equivalently, prices are set so that revenues cover not only current operating expenses but also an interest charge on the book value of the firm's operating assets.²

A frequently voiced claim in the industrial organization literature is that RoR regulation is inherently inefficient. Specifically, product prices are predicted to be too high relative to the levels that a welfare maximizing planner would choose. The alleged reason is that prices must cover the firm's average cost, including

fixed costs, which frequently account for a large fraction of total costs. As a consequence, RoR regulation is not in a position to implement the prices and quantities that correspond to marginal cost pricing (see, e.g., Baumol 1971, Nicholson 2005, Schotter 2008).³

Studies by Biglaiser and Riordan (2000) and Rogerson (2011) have examined RoR regulation in dynamic settings in which the firm makes a sequence of overlapping capacity investments. The significance of multiperiod investment settings is that it becomes possible to determine a product's long-run marginal cost, which includes the cost of providing production capacity for a particular period.⁴ To the extent that the regulated firm's fixed costs consist mainly of capacity-related depreciation charges, the efficiency of RoR regulation then hinges on how well historical cost reflects long-run marginal cost. Rogerson (2011) provides a benchmark result by identifying a particular depreciation schedule with the property that capital costs—i.e., the sum of depreciation and interest charges on the remaining book value of the assets—are indeed equal to the long-run marginal

¹ When government agencies procure unique items, such as weapon systems, prices are frequently calculated on a cost-plus basis, where the mark-up over cost is again calculated to give the contractor a fair return on its invested capital; see, for instance, Laffont and Tirole (1993).

² In certain industries, such as telecommunications, traditional RoR regulation has increasingly been replaced with so-called price cap regulation. Accordingly, prices are set periodically to meet a current RoR constraint. In subsequent years, the regulated firm is then free to charge lower prices until such time as a new price cap is calculated according to an updated RoR calculation (Laffont and Tirole 2000).

³ Parts of the regulation literature have also investigated inefficiencies related to distortions in the regulated firm's input mix (e.g., Averch and Johnson 1962).

⁴ Arrow (1964) shows that despite the inherent joint cost of acquiring assets with a useful life of T periods, in a setting with overlapping investment decisions it is possible to calculate the marginal cost of providing one unit of capacity for one period of time.

cost in each period. This depreciation schedule must not only reflect the anticipated decay of an asset's productive capacity but also anticipated changes in future asset acquisition costs. Falling acquisition costs caused by technological progress are an essential feature of some regulated industries such as telecommunications. Rogerson refers to the corresponding intertemporal cost allocation as the *relative replacement cost* (RRC) rule.

Our analysis in this paper departs from the observation that straight-line depreciation is commonly used by regulatory agencies (Biglaiser and Riordan 2000). This approach is, of course, consistent with the financial reporting practices that most unregulated firms use in connection with property, plant, and equipment. The main contribution of this paper is to identify the dynamic properties of the RoR regulation process when the depreciation rules are not calibrated to the underlying fundamentals, as required in Rogerson's (2011) benchmark result.

For a given depreciation schedule, we find that there is at most one equilibrium candidate for the product price that could emerge asymptotically as the limit of the RoR regulation process. Put differently, product prices could not possibly converge to any other price over time. The unique equilibrium price candidate is lower (higher) than the firm's long-run marginal cost whenever the underlying depreciation schedule for capacity assets is more accelerated (decelerated) than the unbiased depreciation rule identified in Rogerson (2011). In particular, straight-line depreciation amounts to a more accelerated schedule, i.e., assets are written off more quickly early on, if the underlying asset productivity conforms to the *one-hoss shay* ("sudden death") pattern, where productive capacity is undiminished for the entire useful life of the asset. On the other hand, Biglaiser and Riordan (2000) argue that the depreciation policies used in regulatory practice may exhibit deceleration biases insofar as assets are kept on the books beyond their economic useful life. Our results speak to accounting biases in either direction.

An equilibrium price below the long-run marginal cost may seem at odds with the requirement that the regulated firm break even under RoR regulation. It must be kept in mind, though, that the break-even constraint is applied in terms of the firm's residual income. It is well known that for a firm with no initial assets in place the present value of all operating cash flows is equal to the present value of all residual incomes, regardless of the accounting rules (Hotelling 1925, Preinreich 1938).⁵ Given an accelerated (front-loaded) depreciation schedule, the resulting product

prices will exceed long-run marginal cost early, before prices fall below that value and possibly settle at the long-run equilibrium price. Thus, the firm breaks even in each period in accounting terms and does so in economic terms over the entire (infinite) horizon. Of course, the temporal distortions in product prices either above or below the long-run marginal cost do not cancel each other but instead amount to consumer welfare losses.

Our results show that the deviation between the long-run marginal cost and the unique price that can potentially emerge as an equilibrium of the RoR process is not only determined by the choice of depreciation schedule but also by the growth rate in the product market. Specifically, we find that an expanding external product market tends to close the gap between the long-run marginal cost and the potential equilibrium price.⁶ To quantify these distortions, we find that for a stationary product market, the long-run marginal cost of capacity exceeds the historical cost of capacity by about 20% when capacity assets have an undiminished useful life of 25 years and depreciation is calculated according to the straight line rule. Depending on the initial state of the system, the deviation between historical cost and marginal cost may be much larger over long periods of time until product prices approach their equilibrium values.

One might expect the RoR process to be stable in the sense that for a "reasonable" class of starting points prices do indeed converge to the unique equilibrium candidate. Although we have not observed any examples to the contrary, such global stability turns out to be difficult to establish. For a fairly broad class of environments we establish a local stability result that shows that, starting from an initial configuration of past investments for which prices are "close" to the candidate equilibrium price, the resulting trajectory of RoR regulation prices will indeed converge.

Global stability is shown to obtain in our model for a different class of environments in which assets have an unbounded useful life and both capacity decay and accounting depreciation follow a geometric pattern. For this class of "geometric decay" environments, one also obtains monotonic convergence: for a firm that starts out with no assets, prices are above long-run marginal cost in early periods, provided the depreciation schedule is accelerated relative to the RRC rule. Prices then decline monotonically and approach the unique equilibrium price below marginal cost.

In terms of prior work, our analysis is most closely related to Biglaiser and Riordan (2000), Friedl (2007), and Rogerson (2008, 2011). At first glance, some of our findings may seem to contradict those in Friedl

⁵ This identity is sometimes referred to as the *conservation property* of residual income. In a regulation framework, this identity was also observed by Schmalensee (1989).

⁶ This finding applies only to accelerated depreciation schedules. Growth has the opposite effect with decelerated schedules.

(2007), who concludes that RoR regulation will result in prices that are inefficiently high even though the regulator applies straight-line depreciation and the one-hoss shay assumption is being imposed. The key difference is that Friedl (2007) allows for only a single capacity investment decision, the acquisition cost of which is then amortized over a finite horizon. Biglaiser and Riordan (2000) focus their analysis on a setting in which capacity decays geometrically and book values also follow a geometric sequence. Their Theorem 2 concludes that even if the depreciation schedule is unbiased, product prices will eventually exceed the long-run marginal cost. To reconcile this finding with our results, it must be kept in mind that one distinguishing feature of their model is that, because of technical progress, incumbent assets become ultimately obsolete even though they are still functional. Under what Biglaiser and Riordan term “naive” regulation, economically obsolete assets remain part of the rate base, and this effectively creates a bias in the direction of decelerated accounting.

2. Capacity Investments and Product Prices

We consider a single-product firm that makes its capacity investments and pricing decisions subject to a RoR regulation constraint. In each period, the output of the firm, q_t , is bounded by the total capacity available in that period, K_t . The firm is required to operate at capacity, so that $q_t = K_t$.⁷ Production capacity is generated by assets, which have a useful life of T periods. Specifically, an investment expenditure of $v \cdot I_t$ at date t will add capacity to produce an additional $x_\tau \cdot I_t$ units of output at date $t + \tau$, where $1 \leq \tau \leq T$. Thus, new investments come “online” with a lag of one period, yet their productivity may diminish over time. Thus, we assume that $1 = x_1 \geq x_2 \geq \dots \geq x_T > 0$. In the so-called one-hoss shay (or sudden death) scenario all x_τ are equal to one, as the asset is assumed to have undiminished productive capacity over its entire useful life.

At any date $t > T$, the current *state*

$$\boldsymbol{\theta}_t = (I_{t-T}, \dots, I_{t-1})$$

determines the available capacity:

$$K(\boldsymbol{\theta}_t) = I_{t-T} \cdot x_T + \dots + I_{t-1} \cdot x_1.$$

Let $P_t(K_t)$ denote the price that consumers are willing to pay in period t if K_t units of output

are supplied. The functions $P_t(\cdot)$ are assumed to be continuous and decreasing such that $P_t(K_t) \rightarrow 0$ as $K_t \rightarrow \infty$. Furthermore, total revenue $R_t(K_t) = P_t(K_t) \cdot K_t$, is assumed to be concave in K_t .

In terms of costs, the regulated firm is allowed to set product prices to recover both variable operating costs and capital costs, the latter composed of depreciation charges and interest charges imputed on the book value of the firm’s capacity assets. The variable operating cost per unit of output is assumed to be constant and is denoted by w . Assets are capitalized in their acquisition period and then fully depreciated over their useful life, according to some schedule $\mathbf{d} = (d_1, \dots, d_T)$, satisfying $\sum_{\tau=1}^T d_\tau = 1$. Given the current state $\boldsymbol{\theta}_t$, the total depreciation charge in period t is given by

$$D_t(\boldsymbol{\theta}_t) = v \cdot (d_T \cdot I_{t-T} + \dots + d_1 \cdot I_{t-1}).$$

Because depreciation is the only accrual in our model, income is given by revenues less variable operating costs and depreciation:

$$\text{Inc}_t = P_t(K(\boldsymbol{\theta}_t)) \cdot K(\boldsymbol{\theta}_t) - w \cdot K(\boldsymbol{\theta}_t) - D_t(\boldsymbol{\theta}_t).$$

The book value of one capacity unit acquired at date t is originally recorded at its cost v and then amortized over the next T periods according to the depreciation schedule, \mathbf{d} . Specifically, the remaining book value at date $t + \tau$, $0 \leq \tau \leq T$, becomes

$$bv_\tau = \left(1 - \sum_{i=1}^{\tau} d_i\right) \cdot v.$$

Given the current state, $\boldsymbol{\theta}_t$, the aggregate value of capacity assets at date $t - 1$ is given by

$$\text{BV}_{t-1}(\boldsymbol{\theta}_t) = bv_{T-1} \cdot I_{t-T} + \dots + bv_0 \cdot I_{t-1}.$$

Capital costs, which are the sum of depreciation and an imputed charge on the beginning-of-period book value, will be denoted by C_t :

$$C_t(\boldsymbol{\theta}_t) = D_t(\boldsymbol{\theta}_t) + r \cdot \text{BV}_{t-1}(\boldsymbol{\theta}_t),$$

where r denotes the cost of capital.⁸ The constraint imposed by RoR regulation is usually represented in terms of the firm’s accounting rate of return on assets, that is,

$$\frac{\text{Inc}_t}{\text{BV}_{t-1}} \leq r. \quad (1)$$

It has long been recognized in both the regulation and the accounting literature that a firm operating consistently under the constraint imposed by (1) will

⁷ In many industries, such as electricity, there is a natural peak-load pattern in demand so that full capacity utilization effectively amounts to having $q_t = \Delta \cdot K_t$ for some $\Delta < 1$. The presence of peak-load demand does not alter the conclusions obtained in our model in a substantive manner.

⁸ The capital charge rate, r , will generally be determined as a weighted average of the costs of equity and debt. Our analysis treats this cost as exogenous and time invariant.

not make any positive economic profits in the sense that the present value of all cash flows is nonnegative. This follows from the *conservation property of residual income*, which states that, for a firm with no assets at its inception, the present value of cash flows is equal to the present value of all residual incomes, regardless of the applicable depreciation rules. Because the rate of return constraint in (1) is equivalent to

$$RI_t = Inc_t - r \cdot BV_{t-1} \leq 0,$$

it follows that the firm will break even over its entire life time (in terms of discounted cash flows) only if the inequality constraint in (1) is met as an equality in *every* period. Residual income is, by definition, equal to revenues less current operating expenses and less current capital costs, so the firm is instructed to make its investment decision in period t (at date $t - 1$) so that if it sells the capacity available at date t , its revenue will cover the full historical cost. Accordingly, a state θ_t is said to be *feasible* under the RoR regulation if

$$RI_t(\theta_t) \equiv Inc_t(\theta_t) - r \cdot BV_{t-1}(\theta_t) = 0. \quad (2)$$

There may be multiple nonnegative investment levels at date t which lead to a feasible state, θ_{t+1} . For the purposes of our analysis, it does not matter how the firm chooses among these alternative investment levels. On the other hand, depending on the history θ_t and the depreciation rules in place, there may not be an investment level $I_t > 0$ at date t that makes θ_{t+1} feasible. In that case, it must be that $RI_{t+1}(\theta_{t+1}) < 0$ for all $I_t > 0$.⁹ Our analysis takes the perspective that the investment levels up to date T are exogenously given and that the firm is subject to the RoR constraint from date T onward. In particular, new investment levels must be chosen to maintain feasibility. If no such investment level exists, the firm is required to set $I_t = 0$. This rule does not only have intuitive appeal, but, as argued below, setting $I_t = 0$ when feasibility is not attainable in the short run ultimately ensures that feasibility will be restored and maintained in the long run.¹⁰ We refer to an infinite sequence of investments levels, $(I_0, I_1, \dots, I_t, \dots)$, as a *trajectory* θ .

DEFINITION 1. A trajectory θ is compatible with RoR regulation if for all $t \geq T$, $I_t > 0$ implies θ_{t+1} is feasible, i.e., $RI_{t+1}(\theta_{t+1}) = 0$.

⁹ If the sign of $RI_{t+1}(\theta_{t+1})$ were to vary as I_t moves from 0 to large values, then by continuity of the function $RI_{t+1}(\cdot)$ there would exist an I_t where $RI_{t+1}(\cdot)$ is zero. To see that it is impossible to have $RI_{t+1}(\cdot) > 0$ for all $I_t > 0$, we note that $P_t(K_t)$ tends to zero for large values of K_t (induced by high values of I_t). Yet the average cost C_t/K_t tends to a positive number, $w + v \cdot (d_1 + r)$ as I_t gets large.

¹⁰ If feasibility cannot be attained in a given period, the firm may still be able to achieve zero residual income by raising the product price and idling parts of the current capacity. This specification, however, is of no importance for the analysis in this paper.

RoR regulation is usually positioned as an attempt to maximize consumer welfare subject to the constraint that the firm breaks even. Leaving regulatory issues aside, we first characterize the efficient investment and capacity levels that a social planner would choose so as to optimize consumer welfare net of production costs. Consumer welfare at date t is given by

$$B_t(q_t) = \int_0^{q_t} P_t(u) du.$$

Let $\gamma \equiv 1/(1+r)$. Assuming that at date 0 the firm has no assets in place, i.e., $q_0 = 0$ and $q_1 = I_0$, the regulator's welfare maximization problem then becomes¹¹

$$\max_{\{q_t; I_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} [B_t(q_t) - w \cdot q_t - v \cdot I_t] \cdot \gamma^t \quad (3)$$

subject to

$$q_t \leq K_t \equiv I_{t-T} \cdot x_T + \dots + I_{t-1} \cdot x_1.$$

A central insight in the papers of Arrow (1964) and Rogerson (2008, 2011) is that this dynamic maximization problem is intertemporally separable if market demand keeps expanding over time and therefore capacity should be added in each period. This notion is formalized in the following assumption.

ASSUMPTION 1. For any $q > 0$, the inverse demand curves satisfy

$$P_t(q) \leq P_{t+1}(q).$$

Assumption 1 will be maintained throughout our analysis. Intuitively, this assumption ensures that the welfare planner never ends up with excess capacity if in each period the investment is chosen to be myopically optimal to balance current welfare and costs. The following insight dates back to Arrow (1964).

LEMMA 0. The optimal capacity levels, K_t^* , that solve the optimization problem in (3) are given by

$$P_t(K_t^*) = w + c,$$

where

$$c = \frac{v}{\sum_{i=1}^T x_i \cdot \gamma^i}. \quad (4)$$

This result shows that the planner's optimization problem is intertemporally separable. Capacity investments entail an inherent jointness that results from the fact that v dollars spent today generate a T -period stream of future capacity levels. Yet if new

¹¹ As will become clear, it is of no importance to the following derivation that there are no productive capacity assets in place at date 0. It only matters that the process does not commence with excess capacity.

capacity can be added in each period, the cost c effectively becomes the marginal cost of one unit of capacity made available for one period of time. It is also useful to note that c is exactly the price that a hypothetical supplier would charge for renting out capacity for one period of time, if the rental business is constrained to make zero economic profit. Accordingly, we will also refer to c as the *competitive rental price of capacity*.¹²

Lemma 0 suggests that the efficiency of RoR regulation hinges on how well the capital cost $C_t(\theta_t)$ approximates the economically relevant cost $c \cdot K(\theta_t)$. Clearly, the goodness of this approximation depends on the choice of depreciation rules. Rogerson (2008, 2011) provides an important insight in this regard by focusing on the capital costs of an individual project over time. Let

$$z_\tau \equiv v \cdot d_\tau + r \cdot bv_{\tau-1}, \quad (5)$$

so that $z_\tau \cdot I_{t-\tau}$ is the capital cost charged in period t for investment $I_{t-\tau}$ undertaken at date $t - \tau$. Because an investment in one unit of capacity undertaken at date $t - \tau$ still contributes x_τ units of capacity at date t , the economically relevant capital cost at date t is

$$z_\tau^* = x_\tau \cdot c. \quad (6)$$

We refer to the cost allocation rule in (6) as the *relative practical capacity* (RPC) rule, reflecting that the acquisition cost of a new asset is v and that the cost charge in period t is given by the proportion of current capacity relative to total discounted capacity:¹³

$$z_\tau^* = v \cdot \frac{x_\tau}{\sum_{i=1}^T x_i \cdot \gamma^i}.$$

Earlier literature has shown that there is a one-to-one relation between depreciation schedules $\mathbf{d} = (d_1, \dots, d_T)$ and the capital cost charges (z_1, \dots, z_T) . Formally, the linear mapping defined by (5) is one-to-one: for any intertemporal cost charges (z_1, \dots, z_T) , with the property that $\sum_{\tau=1}^T z_\tau \cdot \gamma^\tau = v$, there exists a unique depreciation schedule \mathbf{d} such that (5) is satisfied. We denote by \mathbf{d}^* the unique depreciation schedule corresponding to the RPC cost allocation rule.¹⁴ It is readily verified that in the one-hoss

shay scenario ($x_\tau = 1$), the corresponding depreciation schedule is the *annuity depreciation* method, where $d_\tau = (1 + r) \cdot d_{\tau-1}$. The RPC rule does coincide with *straight-line depreciation* if the productive capacity of assets declines linearly over time such that $x_\tau = 1 - (r/(1 + r \cdot T)) \cdot (\tau - 1)$. Finally, if $T = \infty$ and productive capacity declines geometrically such that $x_\tau = (1 - \alpha) \cdot x_{\tau-1}$ for $0 < \alpha < 1$, the corresponding RPC depreciation schedule is also geometric with $d_\tau = (1 - \alpha) \cdot d_{\tau-1}$.

If depreciation were calculated according to the RPC depreciation rule, the sum of current operating and capital costs would reflect precisely the economically relevant cost that a social planner should impute at date t :

$$\begin{aligned} w \cdot K(\theta_t) + D_t^*(\theta_t) + r \cdot BV_{t-1}^*(\theta_t) \\ &= w \cdot K(\theta_t) + z_T^* \cdot I_{t-T} + \dots + z_1^* \cdot I_{t-1} \\ &= w \cdot K(\theta_t) + c \cdot x_T \cdot I_{t-T} + \dots + c \cdot x_1 \cdot I_{t-1} \\ &= (w + c) \cdot K(\theta_t). \end{aligned}$$

To state Rogerson's (2011) benchmark result in the context of our model, it will be convenient to consider a setting where the firm started with no capacity investments at date 0. Thus, $q_1 = I_0$, $q_2 = I_1 + x_2 \cdot I_0$, and so forth. A trajectory θ is said to be feasible for $t \geq T$ if the corresponding sequence of states $\theta_t = (I_{t-T}, \dots, I_{t-1})$ is feasible.

PROPOSITION 0. *Suppose $K_0 = 0$. If assets are depreciated consistently according to the RPC rule, then RoR regulation results in a trajectory θ that is feasible and attains marginal cost pricing, i.e., $P_t(K(\theta_t)) = c + w$ for all $t \geq 1$.*

This result dispels the economic logic articulated in some microeconomics textbooks (e.g., Baumol 1971, Nicholson 2005, Schotter 2008) regarding the efficiency of RoR regulation. The conventional argument is that because direct transfers to the regulated firm are politically undesirable, regulatory agencies cannot implement marginal cost pricing. For the firm to break even, prices must therefore cover average costs, which include the (historical) fixed costs associated with the regulated firm's investments in property, plant, and equipment. Proposition 0 shows that with suitably chosen depreciation rules, the historical cost of sunk asset expenditures aligns precisely with the long-run marginal cost of capacity.¹⁵

¹² In Biglaiser and Riordan (2000), technical progress causes the long-run marginal cost to fall over time. Accordingly, the acquisition cost of new assets decreases over time and the variable cost of operating successive vintages of capital is assumed to decline.

¹³ Rogerson (2011) refers to the cost allocation rule in (6) as the RRC rule. In the context of his model, the concept of replacement cost is more complex than ours because the acquisition cost of new assets declines exponentially over time.

¹⁴ The RPC rule is closely related to the so-called *relative benefit depreciation* rule, which has played a key role in earlier studies on managerial incentives for investment decisions (see, e.g.,

Reichelstein 1997, Rogerson 1997, Baldenius and Ziv 2003, Bareket and Mohnen 2007, Pfeiffer and Schneider 2007). As the name suggests, the charges under the relative benefit depreciation rule apportion the initial investment expenditure in proportion to the subsequent expected future cash inflows; the RPC rule, in contrast, refers only to the decay in production capacity over time.

¹⁵ Consistent with this finding, Biglaiser and Riordan (2000, p. 752) also observe that "in theory, rate of return regulation could track optimal prices."

Although we regard Proposition 0 as an important benchmark, the result does not speak to the efficiency of RoR regulation if the depreciation rules differ from the RPC rule. Consistent with the financial reporting practice of nonregulated firms, regulatory practice appears to calculate depreciation charges commonly according to the straight-line rule. As observed above, straight-line depreciation is consistent with the RPC rule in the exceptional case where the x_t decline linearly at a specific rate. The natural question for our analysis therefore concerns the dynamic of the rate of regulation process and the types of distortions—both in terms of direction and magnitude—one should expect to see over time as a consequence of using a depreciation policy like the straight-line rule.

3. Stationary Prices

This section examines the set of prices that can emerge as limit points of a trajectory generated by the RoR regulation process. In particular, we seek to characterize how the prices that emerge as stationary values of the RoR process depend on the accounting rules underlying the regulation process and growth in market demand. For reasons of tractability, we restrict attention to so-called *proportionate growth* requiring that

$$P_{t+1}((1 + \mu_t) \cdot K_t) = P_t(K_t). \quad (7)$$

The parameter μ_t then represents the growth rate of the market in period t . We assume $\mu_t < r$. We note that the proportionate growth assumption will be met by constant elasticity demand curves if the elasticity factor remains constant, yet the size of the market expands. The proportionate growth assumption can also be met by linear demand curves of the form $P_t = a_t - b_t \cdot q_t$ if the intercept a_t remains constant, yet the slope parameter b_t contracts over time at the rate $(1 + \mu_t)^{-1}$. As a consequence, demand will become less elastic over time. We note in passing that if an unregulated monopolist has constant marginal (long-run) cost and faces a demand curve exhibiting proportionate growth, the profit-maximizing capacity levels will grow precisely at the rate μ_t in period t . Similarly, in the context of Lemma 0, a social planner would ideally choose capacity levels that expand at the rate μ_t .

DEFINITION 2. A price p_∞ is a stationary price of the RoR regulation process if there exists a trajectory θ that is compatible with RoR regulation and

$$\lim_{t \rightarrow \infty} P_t(K(\theta_t)) = p_\infty.$$

If a price p_∞ satisfies the criterion in Definition 2, we say that θ supports p_∞ . We also consider the stronger notion of a trajectory θ that attains p_∞ in the sense that for all $t > T$, $p_\infty = P_t(K(\theta_t))$. The following result identifies a particular price level that can be attained, provided demand grows at a constant rate.

PROPOSITION 1. If market demand grows at some constant rate μ , then

$$p^* = w + c \cdot \frac{\sum_{\tau=1}^T (1 + \mu)^{T-\tau} \cdot z_\tau(\mathbf{d})}{\sum_{\tau=1}^T (1 + \mu)^{T-\tau} z_\tau^*} \quad (8)$$

is a stationary price of the RoR regulation process, provided $P_T(0) > p^*$. This price is attained by a feasible trajectory of the form $I_{t+1} = I_t \cdot (1 + \mu)$.

From here on, it will be assumed that the price p^* identified in (8) does satisfy the inequality $P_T(0) > p^*$. Given our assumption of proportionate growth, it then follows that $P_t(0) > p^*$ for all $t \geq T$. We note that the result in Proposition 1 is consistent with the benchmark result in Proposition 0 insofar as p^* is equal to the long-run marginal cost $w + c$ whenever depreciation conforms to the RPC rule. We refer to the fraction

$$\Gamma \equiv \frac{\sum_{\tau=1}^T (1 + \mu)^{T-\tau} \cdot z_\tau(\mathbf{d})}{\sum_{\tau=1}^T (1 + \mu)^{T-\tau} \cdot z_\tau^*} \quad (9)$$

as the *accounting bias*. Clearly, this bias factor is a joint function of the depreciation schedule and the growth rate in the product market, μ . Before we delineate the properties of Γ , we establish the following uniqueness result.

PROPOSITION 2. Suppose market demand grows at a constant rate $\mu \geq 0$. The price p^* , given in (8), is the unique stationary price of the RoR regulation process, provided either (i) the productive capacity of assets decays strictly over time, i.e., $x_t > x_{t+1}$, or (ii) the depreciation schedule, \mathbf{d} , has the property that $z_t > z_{t+1}$ with $z_T > 0$.

It should be noted first that Proposition 2 does not say that prices will converge to p^* under the RoR regulation process. However, the result does show that no price other than p^* could possibly be supported as a limit point of this regulation process. The first of the two sufficient conditions identified in Proposition 2 is not particularly restrictive, though it does rule out the one-hoss shay scenario commonly considered in the regulation literature.¹⁶ The alternative sufficient condition in Proposition 2 is satisfied for many commonly used depreciation schedules, including straight-line depreciation. In that case, the historical cost charges z_t will decrease linearly over time.

The uniqueness result in Proposition 2 points to the centrality of the accounting bias factor Γ in determining the efficiency of the RoR regulation process. A natural first question is whether Γ acts as a mark-up or discount on the long-run marginal cost of capacity, c . To address this question, we consider the following partial ordering on the class of depreciation schedules:

¹⁶ However, it can be shown that if the periodic growth rate μ is strictly positive, then the proof of Proposition 2 can be adapted to cover the one-hoss shay scenario as well.

DEFINITION 3. A depreciation schedule \mathbf{d} is more accelerated than \mathbf{d}' if for any $\tau \leq T - 1$

$$bv_\tau(\mathbf{d}) \leq bv_\tau(\mathbf{d}'). \quad (10)$$

Conversely, \mathbf{d} is more decelerated than \mathbf{d}' if the inequalities in (10) are reversed. An accelerated depreciation schedule is front loaded in comparison to the RPC rule. Therefore, the capital costs z_τ of new investments are too high early in the life cycle of an asset. The following result shows how this bias impacts the unique stationary price under RoR regulation.

PROPOSITION 3. *The stationary price p^* , given in (8), satisfies*

$$p^* \leq w + c,$$

provided the depreciation schedule \mathbf{d} is accelerated relative to the RPC rule. Conversely, $p^ \geq w + c$ if \mathbf{d} is decelerated relative to the RPC rule.*

The proof of this result shows that the accounting bias Γ is less than 1 whenever the depreciation schedule is accelerated relative to the RPC rule, whereas Γ is greater than 1 for a decelerated depreciation rule. At first glance, the result in Proposition 3 appears implausible. How can a regulated firm break even in terms of discounted cash flows over an infinite horizon and, at the same time, consumers obtain the product in equilibrium at a price that is below the social marginal cost $w + c$? The answer is that, provided the firm starts with no capacity at date 0, prices must early on exceed the long-run marginal cost, $w + c$, before they ultimately approach $p^* < w + c$. This point will be illustrated further in the setting examined in §4, where both depreciation charges and capacity decay are assumed to follow a geometric pattern. For prices emerging under RoR regulation to be consistently below $w + c$, the firm would have to start with capacity assets in place, and these would effectively serve as a “subsidy” to the system.¹⁷

We now turn to a further examination of the accounting bias factor Γ . Clearly, this factor is a function of both the depreciation schedule and the rate of growth in the product market, μ . To assess the impact

¹⁷Our finding in Proposition 3 stands in contrast to the results obtained by Friedl (2007). He concludes that if assets are depreciated under the straight-line rule and the productivity pattern corresponds to the one-hoss shay scenario, then the resulting capacity level will be too low and the resulting product prices too high, at least in the early periods, relative to the socially efficient levels. Essential to this result, however, is that the regulated firm is assumed to make only a single capacity investment at date 0. As a consequence, prices never reach a steady state level. In fact, because capital costs fall over time due to straight line depreciation, product prices decrease. Friedl’s model predicts excess consumer demand at the regulated prices in later periods yet assumes that the firm does not undertake new capacity investments.

of μ on Γ , we consider a stronger notion of accelerated depreciation. The notion put forth in Definition 3 is essentially a “second-order dominance” condition referring to the cumulative amount amortized at a particular point in time. The following criterion provides a more stringent “first-order dominance” criterion.

DEFINITION 4. A depreciation schedule \mathbf{d} is uniformly more accelerated (decelerated) than the RPC rule if $z_\tau(\mathbf{d})/x_\tau$ is monotonically decreasing (increasing) in τ .

By definition, the depreciation schedule corresponding to the RPC rule, \mathbf{d}^* , has the property that $z_\tau^* = x_\tau \cdot c$. In contrast, a depreciation schedule that is uniformly accelerated relative to the RPC rule starts out with a ratio $z_1^*/x_1 > c$ and then intersects the value c at some point in time $\hat{\tau} < T$.

PROPOSITION 4. *If depreciation is uniformly more accelerated (decelerated) than the RPC rule, the stationary price p^* is increasing (decreasing) in μ .*

To further illustrate Propositions 3 and 4, we provide a numerical characterization of the bias factor Γ in a scenario where depreciation is held fixed at the straight-line rule. The productivity factors x_τ are assumed to decline linearly over time, so that $x_\tau = 1 - \beta(\tau - 1)$. Therefore, $\beta = 0$ corresponds to the one-hoss shay scenario. Figure 1 shows the level sets of the function $\Gamma(\mu, \beta | r, T)$ for values of $r = 0.12$ and $T = 25$. As observed earlier, when $\beta = r/(1 + r \cdot T)$, straight-line depreciation amounts to the RPC rule. As a consequence, $\Gamma = 1$ for $\beta = r/(1 + r \cdot T) = 0.03$. Consistent with the above results, we note that $\Gamma(\cdot)$ assumes values below (above) 1 whenever $\beta < (>)0.03$. Furthermore, it can be verified that the depreciation schedule is in fact uniformly accelerated for $\beta \leq r/(1 + r \cdot T)$. As predicted by Proposition 4, we observe that $\Gamma(\cdot)$ is increasing in μ when $\beta < 0.03$.

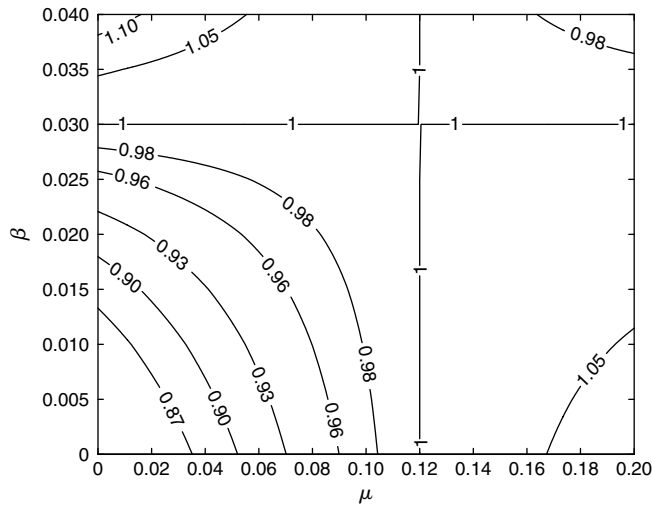
Figure 1 shows that there is no accounting bias if the growth rate in the product market happens to equal the cost of capital, r . This observation follows directly from the observation that

$$\sum_{\tau=1}^T (1+r)^{T-\tau} \cdot z_\tau(\mathbf{d}) = v \cdot \gamma^{-T},$$

for all depreciation schedules \mathbf{d} .¹⁸

With accelerated depreciation (the lower half of the figure), the largest accounting bias occurs in the southwest corner of Figure 1, corresponding to no

¹⁸Directly related to this observation, Rajan and Reichelstein (2009) show that historical cost and marginal cost coincide, regardless of the accounting rules, whenever new investments grow at the firm’s cost of capital.

Figure 1 Level Sets (Isoquants) of the Bias Function $\Gamma(\mu, \beta | r = 0.12, T = 25)$ Under Straight-Line Depreciation

Note. Here, μ represents growth in the product market and β captures the periodic decline in productive capacity.

growth and one-hoss shay. The accounting bias then becomes $\Gamma(0, 0 | r = 0.12, T = 25) = 0.803$. Although this factor may not seem to entail a significant distortion, we recall that it applies only to the asymptotic equilibrium price. To gauge the overall inefficiency associated with accounting biases under RoR regulation, one must consider the losses in the present value of all consumer surpluses resulting from the use of some biased depreciation rule as opposed to the unbiased RPC rule.

4. Stability of the RoR Regulation Process

The preceding characterization of the unique equilibrium price candidate, p^* , raises the following questions about the asymptotic properties of the RoR regulation process. First, is the process locally stable in the sense that it converges if the initial state (and corresponding prices) is sufficiently close to a state that supports p^* ? Second, and more ambitiously, is the RoR process globally stable in the sense that prices will converge to p^* for any initial state?

We are not aware of examples where global stability does not obtain, but we are also not in a position to address the convergence issues at the level of generality maintained in the previous sections. The following analysis will therefore consider specific combinations of productivity patterns and depreciation schedules. We start by assuming that productive capacity follows the one-hoss shay pattern and depreciation is calculated according to the straight-line rule. It is also assumed that demand is stationary over time so that $P_t(\cdot) = P(\cdot)$ for all t . Proposition 2 established

that under these conditions p^* is the unique stationary price. Let $\theta^* = (I^*, I^*, I^*, \dots)$ denote the constant investment trajectory attaining p^* ; i.e., $P(I^* \cdot T) = p^*$. Finally, let $\xi(\cdot, \cdot)$ denote the Euclidean distance in \mathbb{R}^T .

DEFINITION 5. The trajectory θ^* is said to provide a locally stable implementation of p^* if for some $\epsilon > 0$, any trajectory θ that is compatible with RoR regulation θ_T and satisfies $\xi(\theta^*, \theta_T) < \epsilon$ also satisfies

$$\lim_{t \rightarrow \infty} P_t(K(\theta_t)) = p^*.$$

A locally stable implementation requires that if any other trajectory is initially close to θ^* , that trajectory must result in prices that also converge to p^* . We have the following result.

PROPOSITION 5. Given one-hoss shay productivity, straight-line depreciation, and stationary demand, the trajectory θ^* provides a locally stable implementation of p^* .

The proof of Proposition 5 principally relies on the so-called Jury stability criteria, which provide sufficient conditions for local stability.¹⁹

For the remainder of this section, we consider a setting that will be referred to as the *geometric setting*. Assets are infinitely lived and both productive capacity and accounting book values decay geometrically. Formally, the geometric setting satisfies the following specifications:

1. The price in the product market is a linear function of firm output (or capacity). The intercept, a , is stable over time, and the slope declines over time at the rate $(1 + \mu)^{-1}$, where μ is the constant rate at which the overall market grows.²⁰ Formally,

$$P_t(q) = a - b_t \cdot q,$$

where $b_t = b/(1 + \mu)^t$, $\mu \geq 0$.

2. Assets live for infinitely many periods, but the productive capacity of any investment declines geometrically over time at rate α , with $0 < \alpha < 1$. As before, we normalize x_1 to equal 1. Productivity in future periods is then obtained recursively as

$$x_{\tau+1} = (1 - \alpha)x_\tau.$$

3. Depreciation follows a declining-balance rule, represented by a parameter δ , $0 < \delta < 1$. Thus, depreciation expense in each period is a constant proportion δ of the beginning of the period book value. This

¹⁹ The proof of this result is long and tedious. It has therefore been relegated to a separate online companion paper (Nezlobin et al. 2011).

²⁰ This is consistent with the proportionate growth assumption made earlier and implies that demand becomes less elastic over time. As indicated in connection with Proposition 6, some of our results in this section can also be obtained with a constant elasticity demand curve.

implies that the book value of one unit of capacity acquired at date 0 equals the acquisition cost, v , and the remaining book value at any future date $\tau \geq 0$, is given by

$$bv_\tau = v \cdot (1 - \delta)^\tau.$$

Given this specification, the relationship between the depreciation rate and beginning book value can be expressed as

$$d_{\tau+1} = \delta \cdot \frac{bv_\tau}{v}.$$

Taken together, these assumptions imply that book values, depreciation charges, and capital costs all follow a geometric pattern over time: $bv_{\tau+1} = (1 - \delta)bv_\tau$, $d_{\tau+1} = (1 - \delta)d_\tau$ and therefore $z_{\tau+1} = (1 - \delta)z_\tau$.

If investments remain productive indefinitely rather than have a finite life of T years, the definition of a state needs to be modified. In particular, the relevant history includes not just investments made in the prior T periods, but the entire vector of new investments from the inception of the firm. Accordingly, we redefine θ_t as $\theta_t = (I_0, \dots, I_{t-1})$. Note that, because of the geometric structure of the model, the future capacity levels derived from past investments are completely defined by the current capacity level, K_t , and the same observation holds for the capital costs, C_t :

$$C_t(\theta_t) = \sum_{\tau=1}^t z_\tau \cdot I_{t-\tau}.$$

In the geometric setting, current capacity and current capital costs are a “sufficient statistic” for the entire investment history. We again call a state θ_t feasible if the regulation constraint is satisfied as an equality in that state, i.e., $RI_t(\theta_t) = 0$. It is readily verified that in the geometric setting the RPC rule amounts to $\delta = \alpha$; i.e., the depreciation rate matches the rate of decay in productivity. Furthermore, a value $\delta > \alpha$ corresponds to an accelerated depreciation rule.

In the more general settings examined in §§2 and 3, a technical concern was the possibility that there might exist infeasible states along the trajectory of investments, i.e., periods in which no positive investment level would satisfy the rate of return constraint. We show next that this issue does not arise in the geometric decay setting. Starting with a feasible state, there exists one, and only one, positive investment level that meets the following period’s RoR constraint as an equality.

LEMMA 1. *In the geometric setting, suppose depreciation is more accelerated than the RPC rule. Starting from any feasible θ_t , there exists a unique $I_t > 0$ such that θ_{t+1} is feasible.*

We also note that if the firm starts with no capacity assets at date 0, there will be a unique I_0 such that θ_1 is feasible. By induction, it follows then that when a

firm enters the market with no assets in place, there is a unique chain of future investment levels that lead to period-by-period feasibility under RoR regulation.

We now compute the marginal cost of capacity, c , and the capacity level corresponding to p^* in the geometric case. Applying Equation (4) and recognizing that $T = \infty$, the marginal cost of capacity is equal to

$$c = \frac{v}{\sum_{\tau=1}^{\infty} (1 - \alpha)^{\tau-1} \gamma^\tau} = v(\alpha + r).$$

The unique equilibrium price p^* is

$$p^* = w + c \cdot \frac{r + \delta}{r + \alpha} \cdot \frac{\sum_{\tau=1}^{\infty} (1 + \mu)^{-\tau} (1 - \delta)^{\tau-1}}{\sum_{\tau=1}^{\infty} (1 + \mu)^{-\tau} (1 - \alpha)^{\tau-1}} = w + c \cdot \Gamma,$$

where

$$\Gamma = \frac{(r + \delta) \cdot (\alpha + \mu)}{(r + \alpha) \cdot (\delta + \mu)}.$$

Let K^* denote the capacity level corresponding to p^* at date 0, i.e.,

$$a - bK^* = p^*.$$

Therefore,

$$K^* = \frac{1}{b}(a - w - c \cdot \Gamma).$$

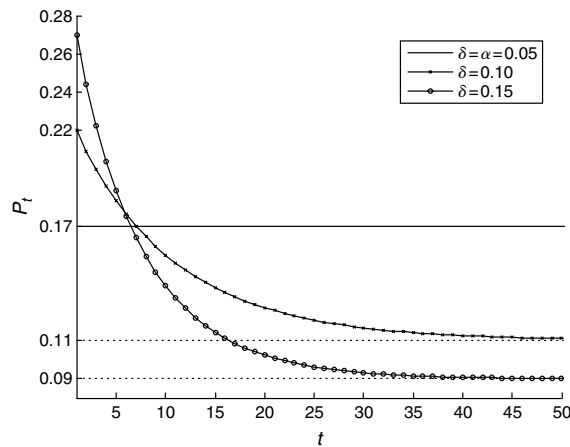
PROPOSITION 6. *In the geometric decay model, suppose depreciation is more accelerated than the RPC rule and the firm starts from a feasible state. Then, as $t \rightarrow \infty$,*

$$\frac{K(\theta_t)}{(1 + \mu)^t} \rightarrow K^* \quad \text{and} \quad P(K(\theta_t)) \rightarrow p^*.$$

Lemma 1 ensures that, starting from any feasible history of prior investments, there is a unique subsequent sequence of positive levels of investment that satisfy the RoR constraint in each period. Proposition 6 shows that this investment trajectory must eventually result in prices converging to the unique equilibrium price, p^* . Moreover, the levels of investment, adjusted for growth, also converge to a unique value, K^* .

As indicated in the Introduction, Biglaiser and Riordan (2000, Theorem 2) establish that under “naive” RoR regulation, prices converge to a value above the long-run marginal cost. To reconcile this finding with our results, including Proposition 6, we note that the specification of naive regulation effectively introduces a bias toward decelerated accounting. By construction, old assets are becoming economically obsolete because of technical progress, which is reflected in decreasing acquisition costs for new assets and declining variable operating costs. Yet the very specification of naive RoR regulation in Biglaiser and Riordan implies that economically obsolete assets remain in the asset base beyond their economically useful life and continue to be depreciated indefinitely. Our result in Proposition 6 can be

Figure 2 Trajectory of Prices Under RoR Regulation for Alternative Depreciation Policies When the Firm Initially Has No Assets in Place



extended to the case of decelerated depreciation, provided the demand curve intercept a is sufficiently far above the stationary price p^* .²¹ In the case of decelerated depreciation, we find that the asymptotic product price is larger than the long-run marginal cost, which is consistent with Biglaiser and Riordan (2000).

The proof of Proposition 6 also identifies the nature of the equilibrium trajectory of investment levels and prices. Specifically, convergence to the equilibrium occurs in a monotonic fashion. Given a feasible state θ_t and the corresponding market price at date t below the equilibrium price, p^* , the unique trajectory of future feasible levels of investment will result in a sequence of prices that are monotonically increasing and converge to p^* .²² On the other hand, if the price is greater than p^* , then every future price will be strictly lower and the sequence will again reach p^* in the limit. Figure 2 illustrates the latter scenario.

If the firm initially has no capacity assets ($K_0 = 0$), the RoR regulation constraint leads to market prices that are above the long-run marginal cost, $w + c$, in early time periods. Ultimately, prices drop below the long-run marginal cost and converge to the equilibrium price, p^* . Figure 2 illustrates this dynamic for the following set of parameters: $w = 0$, $v = 1$, $\mu = 0$, $r = 12\%$, $\alpha = 5\%$, and $P_t(q) = 1 - q$. The horizontal line represents the marginal cost of $v = (\alpha + r) = 0.17$. If depreciation were to conform to the RPC

rule, i.e., if $\delta = \alpha = 0.05$, this price would be realized in every period. The curve corresponding to $\delta = 10\%$ represents an accelerated depreciation method. It intersects the marginal cost line once, from above, and then converges in a strictly decreasing manner to the equilibrium price, 0.11. The curve corresponding to $\delta = 15\%$ depicts the impact of using an even more accelerated depreciation schedule. In this case, the beginning price is greater than before, but the price subsequently drops much faster, cuts the marginal cost line earlier, and eventually converges to the new, lower equilibrium price of 0.09. A natural variant of the example illustrated in Figure 2 is that the firm's initial capacity is 0.9, thereby yielding an initial market price of 0.1. Prices will then increase monotonically and converge to the equilibrium price ($p^* = 0.11$). Thus, prices stay below the marginal cost for the entire duration.

In the general setting of finitely lived assets, Proposition 1 established the existence of a constant growth investment schedule that attains the stationary price, p^* . The argument requires an appropriate initialization in terms of investment choices for the first T periods that can then be continued along a constant growth trajectory. Product prices will then coincide with p^* for all periods from T onward. In the geometric decay setting, a similar sequence can be shown to exist. In fact, it is not necessary to specify the first T investment levels in the geometric scenario. It suffices to identify an initial level of capacity assets and associated historical costs, as well as an appropriate first-period investment. Lemma 1 and the geometric structure then ensure that future investment levels not only maintain feasibility, but also grow at the rate μ . As a consequence, the market price in each subsequent period remains equal to the stationary price, p^* .

From a consumer welfare perspective, one would expect that it is preferable to follow a depreciation schedule with δ closer to α . In particular for the case where $K_0 = 0$, Figure 2 suggests that higher values of δ lead to a price path that exhibits greater variation relative to the underlying long-run marginal costs. Thus, the price distortions tend to be larger both early on and asymptotically. The following result measures *consumer welfare* in accordance with the objective function in (3), i.e., the present value of discounted future welfare levels.²³

NUMERICAL RESULT. Suppose $K_0 = 0$. In the geometric decay model, consumer welfare is monotonically decreasing in δ , provided $\delta \geq \alpha$.²⁴

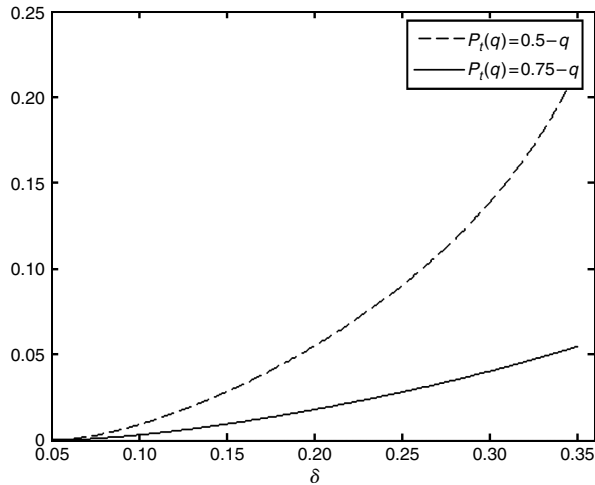
²³ If $K_0 = 0$, the firm breaks even over the entire infinite horizon in the sense that the discounted cash flows (or the discounted residual incomes) sum up to zero.

²⁴ We have not been able to prove this result analytically. Even in a continuous time version of the model, the change in welfare as

²¹ The precise claim and its proof are reported in the online companion paper (Nezlobin et al. 2011).

²² We note that for convergence to be monotonic, the assumption of a linear demand curve seems essential. Numerical analysis indicates that with a constant elasticity demand curve global convergence still obtains; however, prices need not approach p^* in a monotonic fashion. Beyond the geometric decay model, for instance, in the one-hoss shay scenario, it is straightforward to construct examples where prices converge to p^* , albeit not monotonically.

Figure 3 Relative Loss in the Discounted Consumer Welfare as a Function of the Depreciation Rule, δ , for $w = 0$ and $\alpha = 0.05$



This result speaks to the real effects of accounting policy. One of the fundamental issues for capital markets research in accounting is whether investors are misled by accounting rules that do not capture the underlying economics properly. If investors can “see through” the accounting rules and the reported accounting information, the resulting firm valuations and investment decisions may nonetheless be efficient. In the current regulation context, this reasoning obviously does not apply because historical cost is a function of the accounting rules and historical cost directly determines product prices and new investments.

Figure 3 illustrates this result by plotting the percentage welfare loss as a function of the depreciation parameter, δ , under different specifications of the inverse demand function. Both plots assume that $\alpha = 0.05$ and $w = 0$. When the RPC rule is used for regulatory purposes, $\delta = \alpha = 0.05$, RoR regulation results in prices that maximize the discounted consumer welfare, so the relative welfare loss is zero. The two curves in Figure 3 show that as the depreciation schedule becomes more conservative, i.e., as δ increases, a greater share of the consumer welfare is lost. The dashed line represents a relatively smaller market (at any given price point), and the discounted welfare is more sensitive to changes in δ in that case.

a function of δ leads to an intractable expression. However, we have conducted an extensive grid search in Matlab to verify the monotonicity of welfare in δ . Assuming first that $\mu = 0$, we allowed for arbitrary values of v and w and performed the grid search for $0.01 \leq r \leq 0.4$; $w + rv + 0.1v \leq a \leq w + rv + 100v$; $0.1v^2 \leq b \leq 10v^2$ and $0 \leq \alpha \leq \delta \leq \min(1, (a - w)/v - r)$. Finally, it can be shown that if the claim is true for $\mu = 0$, it must then also hold for positive μ ; see the auxiliary lemma in the proof of Lemma 1 in the appendix.

5. Concluding Remarks

This paper has examined the impact of accounting rules on product prices and capacity investments that emerge under RoR regulation. We find that there exists a unique candidate stationary value to which product prices may converge over time. The relation between this stationary price and the firm’s long-run marginal production cost depends on whether the depreciation schedule for fixed assets is accelerated or decelerated relative to the benchmark of neutral accounting. Current regulatory practice appears to favor straight-line depreciation, which represents a form of accelerated depreciation if the productivity of assets is undiminished over their useful life (one-hoss shay scenario). Our analysis shows that the efficiency distortions of the RoR process do not “wash out” over time, even for stationary environments. On the contrary, the distortions caused by a biased depreciation schedule tend to be larger in stationary, no-growth environments.

Our analysis points to the centrality of the depreciation schedule in assessing the asymptotic properties of the RoR mechanism, yet we are mindful of the fact that our model has ignored a number of other factors that are also likely to be of first-order importance in practice. For instance, this paper has treated the cost of new assets as time invariant. In certain industries, however, input prices have shown predictable patterns over time. For instance, the cost of telecommunications equipment has been falling steadily over the past decades. Rogerson (2011) identifies unbiased depreciation rules that extend the concept of RPC depreciation to settings where asset prices decline geometrically over time. A natural extension of our analysis is to study the intertemporal distortions and the limit behavior of the RoR regulation process if historical cost is calculated according to certain rules, such as straight-line depreciation, while the replacement cost of new assets is declining over time.

Second, our study of stationary prices has relied on the assumption that operating costs are assumed to be time invariant and assets do not become economically obsolete until the end of their assumed useful lives. Technological innovation may make it advantageous to replace older but functional assets. As indicated above, this feature is central to the model of Biglaiser and Riordan (2000).

Finally, this paper has ignored the role of corporate income taxes. RoR regulation is commonly based on net income, i.e., income after taxes. Assets are commonly depreciated faster for tax purposes than for financial reporting purposes. The long-run marginal cost of production as well as the stationary price under RoR regulation will then depend on both the depreciation schedule used for regulatory purposes and the one used for tax purposes (see,

for instance, İşlegen and Reichelstein 2011). In the presence of corporate income taxes, the concept of unbiased accounting will change; i.e., the RPC schedule identified in our analysis will no longer lead to efficient outcomes. Studying issues of price convergence and welfare efficiency in a model with changing asset prices and corporate income taxes would bring the predictions of our analysis a significant step closer to the institutional realities of RoR regulation.

Acknowledgments

The authors are grateful to the editor, an associate editor, and three reviewers, as well as seminar participants at University of California, Berkeley; Columbia University; University of California, Davis; Harvard University; Heidelberg University; Ludwig Maximilian University, Munich; Northwestern University; Peking University; Toronto University; and Yale University for their helpful comments and suggestions. The authors also thank Yanruo Wang for excellent research assistance.

Appendix

PROOF OF PROPOSITION 1. Let K_T be such that $P_T(K_T) = p^*$ and let

$$I = \frac{K_T}{\sum_{\tau=1}^T (1 + \mu)^{T-\tau} x_\tau}.$$

Consider a trajectory, θ , for which investments start at $I_0 = I$ and then grow at rate μ :

$$I_t = (1 + \mu)^t I.$$

For this trajectory, for any $t \geq T$, we have

$$K(\theta_t) = I \cdot \sum_{\tau=1}^T (1 + \mu)^{t-\tau} x_\tau = (1 + \mu)^{t-T} K_T.$$

Therefore,

$$P_t(K(\theta_t)) = P_T(K_T) = p^*.$$

The historical cost in period t is given by

$$H_t(\theta_t) = w \cdot K(\theta_t) + \sum_{\tau=1}^T (1 + \mu)^{t-\tau} z_\tau(\mathbf{d}) \cdot I.$$

Revenues in period t are

$$\begin{aligned} P_t(K(\theta_t)) \cdot K(\theta_t) &= p^* \cdot K(\theta_t) \\ &= w \cdot K(\theta_t) + \frac{\sum_{\tau=1}^T (1 + \mu)^{T-\tau} z_\tau(\mathbf{d})}{\sum_{\tau=1}^T (1 + \mu)^{T-\tau} z_\tau^*} \cdot c \cdot K(\theta_t). \end{aligned}$$

To prove that the trajectory is compatible, it suffices to show that

$$I = \frac{c \cdot K(\theta_t) \cdot (1 + \mu)^{T-t}}{\sum_{\tau=1}^T (1 + \mu)^{T-\tau} z_\tau^*}.$$

But this equality holds because $K(\theta_t) \cdot (1 + \mu)^{T-t} = K_T$ and $z_\tau^* = c \cdot x_\tau$ (see Equation (6)). □

PROOF OF PROPOSITION 2. Assume that

$$\lim_{t \rightarrow \infty} P_t(K(\theta_t)) = p_\infty$$

for some trajectory θ that is compatible with RoR regulation. Let $K_\infty = P_0^{-1}(p_\infty)$. Because prices converge to p_∞ , the growth-adjusted capacity levels must converge to K_∞ :

$$\lim_{t \rightarrow \infty} \frac{K_t}{(1 + \mu)^t} = K_\infty.$$

We establish three results, which, taken together, imply that $p_\infty = p^*$.

STEP A. Under the conditions of Proposition 2, if the trajectory θ results in convergent prices, then this trajectory can have at most a finite number of infeasible states.

PROOF OF STEP A. Assume first that condition (i) of Proposition 2 is met. Let $\theta_{t+1} = (I_{t-T+1}, \dots, I_t)$ be an infeasible state in trajectory θ . Then, it has to be that $I_t = 0$ and

$$K_{t+1} = x_2 I_{t-1} + \dots + x_T I_{t-T+1}.$$

By definition,

$$K_t = x_1 I_{t-1} + \dots + x_T I_{t-T}. \tag{11}$$

Therefore, we have

$$K_{t+1} = K_t - (x_1 - x_2) I_{t-1} - \dots - (x_T - x_{T+1}) I_{t-T}, \tag{12}$$

where we defined x_{T+1} to be zero. All the terms that are subtracted from K_t in the right-hand side of (12) are nonnegative because x_τ s are decreasing. Let

$$\tau_0 = \arg \max_{\tau=1, \dots, T} x_\tau I_{t-\tau}.$$

Equation (11) implies that $x_{\tau_0} I_{t-\tau_0} \geq K_t/T$. Therefore,

$$(x_{\tau_0} - x_{\tau_0+1}) I_{t-\tau_0} \geq \frac{(x_{\tau_0} - x_{\tau_0+1})}{T x_{\tau_0}} K_t.$$

Substituting this inequality into (12), we obtain

$$K_{t+1} \leq K_t - \frac{(x_{\tau_0} - x_{\tau_0+1})}{T x_{\tau_0}} K_t.$$

Let

$$\psi = \frac{1}{2} \min_{\tau=1, \dots, T} \frac{(x_\tau - x_{\tau+1})}{T x_\tau}.$$

The strict monotonicity of the x_τ s implies that ψ is greater than zero. We have shown that

$$K_{t+1} < (1 - \psi) K_t$$

for any infeasible state θ_{t+1} and some constant $\psi > 0$. Because μ is nonnegative and growth-adjusted capacity levels converge to K_∞ , there can only be a finite number of states such that $K_{t+1} < (1 - \psi) K_t$, i.e., only a finite number of infeasible states.

Assume now that condition (ii) of Proposition 2 is met. The preceding arguments establish that

$$C_{t+1} < (1 - \psi) C_t$$

for any infeasible state θ_{t+1} and some positive ψ . Because prices and growth-adjusted capacity levels converge, it must be that

$$\lim_{t \rightarrow \infty} \frac{(P_t(K_t) - w) K_t}{(1 + \mu)^t} = (p_\infty - w) K_\infty.$$

Therefore, for some sufficiently large t_0 and any $t > t_0$

$$(P_{t+1}(K_{t+1}) - w)K_{t+1} > \left(1 - \frac{\psi}{2}\right)(P_t(K_t) - w)K_t.$$

If there are infinitely many infeasible states, then there will exist $t > t_0$ such that θ_t is feasible and θ_{t+1} is not (recall that feasible states happen at least every T periods because $P_0(0) > w + z_1$). For this value of t , we have

$$\begin{aligned} (P_{t+1}(K_{t+1}) - w)K_{t+1} &> \left(1 - \frac{\psi}{2}\right)(P_t(K_t) - w)K_t \\ &= \left(1 - \frac{\psi}{2}\right)C_t > C_{t+1}. \end{aligned}$$

This contradicts the fact that residual income has to be negative in every infeasible state. Therefore, trajectory θ passes only through a finite number of infeasible states. \square

STEP B. Under the conditions of Proposition 2, if p_∞ is a stationary price, then there must exist a feasible trajectory θ' that attains p_∞ , i.e., for all $t \geq T$,

$$P_t(K(\theta'_t)) = p_\infty.$$

PROOF OF STEP B. Recall that $\theta_t = (I_{t-T}, \dots, I_{t-1})$ and consider the following sequence of T -dimensional vectors. Define $\mathbf{s}(t)$ to be the zero vector for $t < T$ and

$$\mathbf{s}(t) = \frac{\theta_t}{(1 + \mu)^t}$$

for $t \geq T$. Letting $\mathbf{s}_\tau(t)$ denote the τ th component of $\mathbf{s}(t)$, we have

$$\mathbf{s}_\tau(t) = \frac{I_{t-T-1+\tau}}{(1 + \mu)^t}.$$

Note that $K(\mathbf{s}(t)) = (1 + \mu)^{-t}K(\theta_t)$. The fact that the market grows at rate μ implies

$$P_0(K(\mathbf{s}(t))) = P_t((1 + \mu)^t K(\mathbf{s}(t))) = P_t(K(\theta_t)).$$

Therefore,

$$P_0(K(\mathbf{s}(t))) \rightarrow p_\infty. \quad (13)$$

Because $P_0(\cdot)$ is monotonic and continuous, the equation above implies

$$K(\mathbf{s}(t)) \rightarrow K_\infty, \quad (14)$$

where $K_\infty = P_0^{-1}(p_\infty)$. Hence,

$$P_0(K(\mathbf{s}(t)))K(\mathbf{s}(t)) \rightarrow p_\infty K_\infty. \quad (15)$$

Step A established that the trajectory θ can have at most a finite number of infeasible states. Therefore, for values of t greater than some t_0 , all states θ_t are feasible. For $t > t_0$, we have

$$\begin{aligned} P_0(K(\mathbf{s}(t)))K(\mathbf{s}(t)) &= (1 + \mu)^{-t}P_t(K(\theta_t))K(\theta_t) \\ &= (1 + \mu)^{-t}H(\theta_t) = H(\mathbf{s}(t)). \end{aligned}$$

Applying (15), we obtain

$$H(\mathbf{s}(t)) \rightarrow p_\infty K_\infty. \quad (16)$$

We now show that the vectors $\mathbf{s}(t)$ are bounded. Recall that

$$K(\mathbf{s}(t)) = x_1 \mathbf{s}_T(t) + \dots + x_T \mathbf{s}_1(t).$$

All x s are strictly greater than zero, so condition (14) implies that $\mathbf{s}_\tau(t)$ are bounded for every τ .

We next construct a trajectory $\theta' = (I'_0, I'_1, \dots, I'_t, \dots)$ such that θ'_t is feasible for every $t \geq T$ and

$$P_t(K(\theta'_t)) = p_\infty$$

for every $t \geq T$. Because all vectors $\mathbf{s}(t)$ are drawn from a bounded set, there must exist a converging subsequence $\mathbf{s}(t_i)$. Let

$$\theta'_T = (1 + \mu)^T \lim_{i \rightarrow \infty} \mathbf{s}(t_i).$$

Applying the linearity of $K(\cdot)$ and Equation (14), we can compute capacity and revenues in state θ'_T as follows:

$$\begin{aligned} K(\theta'_T) &= (1 + \mu)^T K\left(\lim_{i \rightarrow \infty} \mathbf{s}(t_i)\right) = (1 + \mu)^T \lim_{i \rightarrow \infty} K(\mathbf{s}(t_i)) \\ &= (1 + \mu)^T K_\infty, \end{aligned}$$

$$\begin{aligned} P_T(K(\theta'_T))K(\theta'_T) &= P_T((1 + \mu)^T K_\infty)(1 + \mu)^T K_\infty \\ &= (1 + \mu)^T P_0(K_\infty)K_\infty. \end{aligned}$$

The cost function is also linear in investments; therefore,

$$\begin{aligned} H(\theta'_T) &= (1 + \mu)^T H\left(\lim_{i \rightarrow \infty} \mathbf{s}(t_i)\right) = (1 + \mu)^T \lim_{i \rightarrow \infty} H(\mathbf{s}(t_i)) \\ &= (1 + \mu)^T p_\infty K_\infty. \end{aligned}$$

From these equations, one can see that state θ'_T is feasible and $P_T(K(\theta'_T)) = p_\infty$. It remains to show that this trajectory can be continued indefinitely into the future so that state feasibility and prices are preserved. The following substep completes the proof of Step B.

SUBSTEP B1. If state θ'_t is feasible for some t , $P_t(K(\theta'_t)) = p_\infty$, and there exists a subsequence of $\mathbf{s}(t)$, $\mathbf{s}(t_i)$, such that

$$\theta'_t = (1 + \mu)^t \lim_{i \rightarrow \infty} \mathbf{s}(t_i) \quad (17)$$

then there exists an investment I'_t such that state θ'_{t+1} is feasible,

$$P_{t+1}(K(\theta'_{t+1})) = p_\infty,$$

and there exists a subsequence of $\mathbf{s}(t)$, $\mathbf{s}(t'_i)$, such that

$$\theta'_{t+1} = (1 + \mu)^{t+1} \lim_{i \rightarrow \infty} \mathbf{s}(t'_i). \quad (18)$$

PROOF OF SUBSTEP B1. See the online companion paper (Nezlobin et al. 2011). \square

STEP C. Under the conditions of Proposition 2, p^* is the unique stationary price of the RoR regulation process.

PROOF OF STEP C. We will prove this step assuming that condition (i) of Proposition 2 holds. Under condition (ii), the proof is analogous with x_τ replaced by z_τ and K_t replaced by C_t .

If θ supports p^∞ , then, as was shown in the previous step, there exists a trajectory θ' that attains p^∞ . For notational simplicity, we will now redefine θ to be the trajectory that attains p^∞ . First, let us show that there exists some η such that

$$I_t - \eta K_t \rightarrow s^*$$

when $t \rightarrow \infty$, for some constant s^* . To sustain constant prices, the firm needs to grow capacity at rate μ :

$$K_{t+1} = (1 + \mu)K_t.$$

The latest investment can be expressed as

$$I_t = \frac{\mu}{x_1} K_t + \frac{x_1 - x_2}{x_1} I_{t-1} + \dots + \frac{x_T - x_{T+1}}{x_1} I_{t-T}, \quad (19)$$

where $x_{T+1} = 0$. There exists some η satisfying

$$\frac{\mu}{x_1} = \eta - \frac{x_1 - x_2}{x_1(1 + \mu)} \eta - \dots - \frac{x_T - x_{T+1}}{x_1(1 + \mu)^T} \eta. \quad (20)$$

If $\mu = 0$, then any η satisfies the equation above and we will proceed with $\eta = 0$. If $\mu > 0$, then the coefficient on η in the right-hand side is strictly greater than zero and the equation defines a unique (positive) η . Because

$$K_{t-\tau} = \frac{K_t}{(1 + \mu)^\tau},$$

Equations (19) and (20) imply

$$(I_t - \eta K_t) = \frac{x_1 - x_2}{x_1} (I_{t-1} - \eta K_{t-1}) + \dots + \frac{x_T - x_{T+1}}{x_1} (I_{t-T} - \eta K_{t-T}).$$

Let $s_t = I_t - \eta K_t$. We then have the following claim.

SUBSTEP C1. *The sequence $\{s_t\}$ converges to some s^* .*

PROOF OF SUBSTEP C1. See the online companion paper (Nezlobin et al. 2011).

To complete the proof of Step C, we note that if $\mu = 0$, then $s_t = I_t$ and we have shown that investment levels converge. In that case,

$$p_\infty - w = \frac{C(\theta_t)}{K(\theta_t)} = \lim_{t \rightarrow \infty} \frac{C(\theta_t)}{K(\theta_t)} = \frac{s^* \sum_{\tau=1}^T z_\tau}{s^* \sum_{\tau=1}^T x_\tau} = p^* - w.$$

Now assume that $\mu > 0$ and let $K_\infty = P_0^{-1}(p_\infty)$. Then, $K_t = (1 + \mu)^t K_\infty$, and, therefore,

$$K_\infty = \frac{K_t}{(1 + \mu)^t} = \lim_{t \rightarrow \infty} \frac{K_t}{(1 + \mu)^t}.$$

Because $\lim_{t \rightarrow \infty} (I_t - \eta K_t) = 0$,

$$\lim_{t \rightarrow \infty} \frac{I_t - \eta K_t}{(1 + \mu)^t} = 0.$$

Hence,

$$\lim_{t \rightarrow \infty} \frac{I_t}{(1 + \mu)^t} = \eta K_\infty. \quad (21)$$

To compute η , we note that

$$\begin{aligned} K_\infty &= \lim_{t \rightarrow \infty} \frac{K_t}{(1 + \mu)^t} = \lim_{t \rightarrow \infty} \frac{x_1 I_{t-1} + \dots + x_T I_{t-T}}{(1 + \mu)^t} \\ &= \left(\frac{x_1}{(1 + \mu)} + \dots + \frac{x_T}{(1 + \mu)^T} \right) \eta K_\infty, \end{aligned}$$

and, therefore,

$$\eta = \frac{1}{\sum_{\tau=1}^T x_\tau (1 + \mu)^{-\tau}}.$$

Because prices are constant, capacity grows at rate μ , and the RoR constraint is satisfied in every state, capital costs must also grow at rate μ . Let $C_\infty = C_t / (1 + \mu)^t$. Then,

$$C_\infty = \frac{C_t}{(1 + \mu)^t} = \lim_{t \rightarrow \infty} \frac{C_t}{(1 + \mu)^t} = \lim_{t \rightarrow \infty} \frac{z_1 I_{t-1} + \dots + z_T I_{t-T}}{(1 + \mu)^t}.$$

Let us now apply (21):

$$C_\infty = \lim_{t \rightarrow \infty} \frac{z_1 I_{t-1} + \dots + z_T I_{t-T}}{(1 + \mu)^t} = \sum_{\tau=1}^T z_\tau (1 + \mu)^{-\tau} \eta K_\infty.$$

To conclude, observe that

$$p_\infty - w = \frac{C_\infty}{K_\infty} = \frac{\sum_{\tau=1}^T z_\tau (1 + \mu)^{-\tau}}{\sum_{\tau=1}^T x_\tau (1 + \mu)^{-\tau}} = p^* - w.$$

This concludes the proof of Proposition 2. \square

PROOF OF PROPOSITION 3. Define

$$h(\mu, \mathbf{d}) \equiv (1 + \mu)^0 \cdot z_T(\mathbf{d}) + \dots + (1 + \mu)^{T-1} \cdot z_1(\mathbf{d}).$$

By definition,

$$\Gamma \equiv \frac{h(\mu, \mathbf{d})}{h(\mu, \mathbf{d}^*)}, \quad (22)$$

where \mathbf{d}^* is the unique depreciation schedule corresponding to the RPC rule. In particular, $z_t(\mathbf{d}^*) \equiv z_t^*$. To prove the claim, it suffices to show that

$$h(\mu, \mathbf{d}) < h(\mu, \mathbf{d}^*),$$

whenever \mathbf{d} is accelerated relative to \mathbf{d}^* and, conversely, that the inequality is reversed if \mathbf{d} is decelerated relative to \mathbf{d}^* . Because $\mu < r$, both claims follow directly from Proposition 1 in Rajan and Reichelstein (2009). \square

PROOF OF PROPOSITION 4. By construction of the RPC rule,

$$h(\mu, \mathbf{d}^*) = c \cdot [(1 + \mu)^0 \cdot x_T + \dots + (1 + \mu)^{T-1} \cdot x_1],$$

where $h(\cdot, \cdot)$ is the same as in the proof of Proposition 3. Proposition 3 in Rajan and Reichelstein (2009) shows that

$$\frac{(1 + \mu)^0 \cdot z_T(\mathbf{d}) + \dots + (1 + \mu)^{T-1} \cdot z_1(\mathbf{d})}{c \cdot [(1 + \mu)^0 \cdot x_T + \dots + (1 + \mu)^{T-1} \cdot x_1]}$$

is increasing (decreasing) in μ provided $z_t(\mathbf{d})/x_t$ is monotonically decreasing (increasing) in t . That is precisely the definition of a depreciation schedule that is uniformly accelerated (decelerated) relative to the RPC rule. \square

PROOF OF LEMMA 1. First, we will establish an auxiliary lemma that will convert the case of constant market growth to the case of stationary markets ($\mu = 0$). We subsequently prove Lemma 1 under the assumption of no growth.

AUXILIARY LEMMA. *Given a model with constant market growth, consider the following model with stationary markets:*

$$P'_t(K'_t) = a - bK'_t,$$

$$x'_\tau = \frac{x_\tau}{(1 + \mu)^\tau},$$

$$z'_\tau = \frac{z_\tau}{(1 + \mu)^\tau}.$$

Then, state $\theta_t = (I_0, \dots, I_{t-1})$ is feasible in the original constant growth model if and only if state $\theta'_t = (I_0/(1 + \mu)^0, \dots, I_{t-1}/(1 + \mu)^{t-1})$ is feasible in the stationary model defined above.

PROOF. First observe that

$$K'_t(\theta'_t) = \sum_{\tau=1}^t I_{t-\tau}(1+\mu)^{-(t-\tau)} x_\tau (1+\mu)^{-\tau} = (1+\mu)^{-t} K_t.$$

Hence, $P'_t(K'_t) = P_t(K_t)$ and $P'_t(K'_t)K'_t = (1+\mu)^{-t} P_t(K_t)K_t$. It remains to show that

$$H'_t(\theta'_t) = (1+\mu)^{-t} H_t(\theta_t).$$

Indeed,

$$H'_t(\theta'_t) = \sum_{\tau=1}^t I_{t-\tau}(1+\mu)^{-(t-\tau)} z_\tau (1+\mu)^{-\tau} = (1+\mu)^{-t} H_t. \quad \square$$

It remains to establish Lemma 1 in the case of no growth. Since state θ_t is feasible, we have

$$P_t(K_t)K_t = C_t + wK_t. \quad (23)$$

State θ_{t+1} is feasible if and only if

$$P_{t+1}(K_{t+1})K_{t+1} = C_{t+1} + wK_{t+1}. \quad (24)$$

Capital costs in period $t+1$ can be decomposed into the sum of capital costs of investments prior to I_t and the capital cost of the latest investment:

$$C_{t+1} = C_t(1-\delta) + z_1 I_t.$$

Applying (23), the first term in the right-hand side of the equation above can be rewritten as

$$C_t(1-\delta) = (1-\delta)(P_t(K_t)K_t - wK_t).$$

For declining balance depreciation, $z_1 = \delta v + r v$. Because $x_1 = 1$ and capacity declines at the rate $(1-\alpha)$,

$$I_t = K_{t+1} - (1-\alpha)K_t.$$

These observations together imply that state θ_{t+1} is feasible if and only if

$$\begin{aligned} P_{t+1}(K_{t+1})K_{t+1} &= (1-\delta)(P_t(K_t)K_t - wK_t) \\ &\quad + (\delta+r)v(K_{t+1} - (1-\alpha)K_t) + wK_{t+1}. \end{aligned}$$

Given the linearity of demand curves, this equality can be rewritten as

$$\begin{aligned} (a - bK_{t+1})K_{t+1} &= (1-\delta)((a - bK_t)K_t - wK_t) \\ &\quad + (\delta+r)v(K_{t+1} - (1-\alpha)K_t) + wK_{t+1}. \end{aligned}$$

We obtain a quadratic equation in K_{t+1} and it has two solutions:

$$K_{t+1}^{(1)} = \frac{1}{2b}(a - w - v(r+\delta)) + \frac{1}{2b}\sqrt{S(K_t)} \quad (25)$$

and

$$K_{t+1}^{(2)} = \frac{1}{2b}(a - w - v(r+\delta)) - \frac{1}{2b}\sqrt{S(K_t)}, \quad (26)$$

where

$$\begin{aligned} S(K_t) &\equiv (a - w - v(r+\delta))^2 + 4b(-a + r v + w - r v \alpha \\ &\quad + (a + v - w - v \alpha)\delta)K_t + 4b^2(1-\delta)K_t^2. \end{aligned} \quad (27)$$

Let us now recall that $I_t = K_{t+1} - (1-\alpha)K_t$ and compute the corresponding investments:

$$I_t^{(1)} = \frac{1}{2b}(a - w - v(r+\delta) - 2(1-\alpha)bK_t) + \frac{1}{2b}\sqrt{S(K_t)},$$

$$I_t^{(2)} = \frac{1}{2b}(a - w - v(r+\delta) - 2(1-\alpha)bK_t) - \frac{1}{2b}\sqrt{S(K_t)}.$$

First, we will show that $I_t^{(2)}$ is negative, and, therefore, if a positive feasible continuation exists, it is unique. Second, we will show that a positive feasible I_t exists (this will imply that $I_t^{(1)}$ is positive). For the former statement, it suffices to check that

$$(a - w - v(r+\delta) - 2(1-\alpha)bK_t)^2 < S(K_t).$$

Expanding and subtracting the right-hand side from the left-hand side, the inequality above can be rewritten as

$$4bK_t(bK_t(\alpha^2 - 2\alpha + \delta) + (a - w)(\alpha - \delta)) < 0. \quad (28)$$

Let us assume that $\alpha^2 - 2\alpha + \delta > 0$. Then, the left-hand side of (28) is less than zero if

$$K_t < \frac{(a - w)(\delta - \alpha)}{b(\alpha^2 - 2\alpha + \delta)}.$$

Note that $\delta - \alpha > \alpha^2 - 2\alpha + \delta$. Therefore,

$$\frac{(a - w)(\delta - \alpha)}{b(\alpha^2 - 2\alpha + \delta)} > \frac{a - w}{b}.$$

On the other hand, we know that $a - bK_t > w$, which implies $K_t < (a - w)/b$, and inequality (28) has to hold. Now let us assume that $\alpha^2 - 2\alpha + \delta < 0$. Then, inequality (28) holds whenever

$$K_t > \frac{(a - w)(\delta - \alpha)}{b(\alpha^2 - 2\alpha + \delta)}.$$

In that case, because by assumption $\delta > \alpha$ and $P_t(K_t) > w$, we have $a > w$. Therefore,

$$\frac{(a - w)(\delta - \alpha)}{b(\alpha^2 - 2\alpha + \delta)} < 0$$

and K_t has to be greater than this number.

To show that a positive feasible I_t exists, we first observe that, if the company does not invest anything in period t , its total costs will be lower than revenues:

$$\begin{aligned} P_{t+1}(K_{t+1})K_{t+1} &= (1-\alpha)P_{t+1}((1-\alpha)K_t)K_t > (1-\alpha)P_t(K_t)K_t \\ &= (1-\alpha)(C_t + wK_t) > (1-\delta)C_t + (1-\alpha)wK_t \\ &= C_{t+1} + wK_{t+1}. \end{aligned}$$

Yet if the company were to invest $K_{t+1} = a/b$, its revenues would be equal to zero, and its costs would be positive. Therefore, there must exist a positive I_t which equates revenues total costs. \square

PROOF OF PROPOSITION 6. By the same auxiliary argument that was employed in the proof of Lemma 1, the case of constant market growth can be converted to the case of no growth by adjusting x_τ and z_τ appropriately. We therefore restrict attention to the case when $\mu = 0$.

First, we will show that $K_{t+1} > K_t$ if $K_t < K^*$ and $K_{t+1} < K_t$ otherwise. We know that K_{t+1} will be given by the

expression for $K_{t+1}^{(1)}$ from the proof of the previous lemma, Equation (25):

$$K_{t+1} = \frac{1}{2b}(a - w - v(r + \delta)) + \frac{1}{2b}\sqrt{S(K_t)},$$

where $S(K_t)$ is defined by equation (27).

It is clear from this formula that, starting from K_2 , it will be the case that

$$K_t \geq \frac{1}{2b}(a - w - v(r + \delta)).$$

Therefore, K_{t+1} is greater than K_t whenever

$$(a - w - v(r + \delta) - 2bK_t)^2 < S(K_t).$$

This is equivalent to

$$4bK_t(-a + w + v(r + \delta) + b\delta K_t) < 0.$$

The latter inequality holds when

$$0 < K_t < \frac{(a - w)\delta - v\alpha(r + \delta)}{b\delta} = K^*,$$

and the converse holds when $K_t > K^*$. We have shown that $K_{t+1} > K_t$ if $K_t < K^*$ and $K_{t+1} < K_t$ otherwise.

We next show that $K_{t+1} > K^*$ if $K_t > K^*$ and $K_{t+1} < K^*$ otherwise. We first observe that because $\delta > \alpha$,

$$\frac{1}{2b}(a - w - v(r + \delta)) < \frac{(a - w)\delta - v\alpha(r + \delta)}{b\delta} = K^*.$$

Therefore, $K_{t+1} > K^*$ if and only if

$$\left(a - w - v(r + \delta) - 2\frac{(a - w)\delta - v\alpha(r + \delta)}{\delta}\right)^2 < S(K_t).$$

This condition is equivalent to

$$\frac{4((w - a)\delta + v\alpha(r + \delta) + bK_t\delta)(v(\alpha - \delta)(r + \delta) - b(1 - \delta)\delta K_t)}{\delta^2} < 0. \quad (29)$$

The quadratic function in the numerator of the left-hand side has two roots:

$$\left\{ \frac{(a - w)\delta - v\alpha(r + \delta)}{b\delta}, \frac{v(\alpha - \delta)(r + \delta)}{b(1 - \delta)} \right\}.$$

The first root is equal to K^* and the second root is negative because of the assumption $\delta > \alpha$. The coefficient on K_t^2 in (29) is negative, so the function will be greater than zero for $0 < K_t < K^*$ and less than zero for $K_t > K^*$. We have shown that if $K_t < K^*$, then $K_t < K_{t+1} < K^*$, and if $K_t > K^*$, then $K_t > K_{t+1} > K^*$. These inequalities imply that the sequence $\{K_t\}$ converges to some limit K_∞ . Let us show that $K_\infty = K^*$. Because all states θ_t are feasible, $P(K_t)K_t = wK_t + C_t$, and therefore

$$wK_t + C_t \rightarrow P(K_\infty)K_\infty.$$

Because $I_t = K_{t+1} - (1 - \alpha)K_t$, $I_t \rightarrow \alpha K_\infty$. On the other hand, $(\delta + r)vI_t = C_{t+1} - (1 - \delta)C_t$; therefore,

$$(\delta + r)vI_t \rightarrow \delta(P(K_\infty)K_\infty - wK_\infty).$$

We conclude that

$$\alpha K_\infty = \frac{\delta(P(K_\infty)K_\infty - wK_\infty)}{(\delta + r)v}$$

and

$$P(K_\infty) = w + \frac{\alpha(\delta + r)}{\delta}v = p^*. \quad \square$$

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