A Fractional Solution to a Stock Market Mystery[1]

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Abstract

Beginning in early 2021, surging volume in Berkshire Hathaway A (BRK.A) has captured the attention of market watchers. Averaging volume of just 375 shares a day over the previous decade, the market's most expensive stock routinely traded additional volume equivalent to more than a billion dollars a day, trading which continues to this day. We demonstrate that this volume is due to the interaction of a well-intentioned but misguided FINRA reporting rule, Robinhood trading, and fractional shares. We show that this reported phantom volume now represents 80% of BRK.A's daily trading volume and has created dislocations in BRK.A's relationship to its paired stock BRK.B, missed arbitrage opportunities for a confused market, and higher trading costs. We argue that how to incorporate fractional share trading into the national market system is the real mystery, and we provide potential solutions.

JEL classification: G14, G21, G23, G24

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1. Introduction

Since mid-February 2021, mysterious trading in Berkshire Hathaway A (BRK.A) has captured the attention of market watchers. Specifically, on February 18 volume jumped to 1,250 shares, having averaged only 375 shares a day for the prior decade. The surging volume continued, staying above 2,000 shares a day for the next 3 months, with volume on a number of days exceeding 3,000 shares and even hitting over 4,200 shares on March 10. Given Berkshire's status as the most expensive listed U.S. stock, this additional volume represented more than $1 billion of extra trading each day. Figure 1 shows that the daily number of shares transacted in BRK.A are now typically a factor of five or ten times higher than before mid-January 2021. This increased volume in BRK.A was also accompanied by a rising stock price, with one day actually recording a 51% increase in after-hours trading (a price movement that was largely reversed the next day). Adding to the mystery was that Berkshire Hathaway B (BRK.B) exhibited virtually none of these effects, despite the fact that these are "paired stocks"—both have claims to the same underlying corporate profits and historically, the price of BRK.A hovers around 1,500 times that of BRK.B. What was going on here??

[Figure 1 about here]

Explanations flew fast and furious. Baron's favored a "mystery buyer" explanation in their two articles on the topic, confident that this would be confirmed by upcoming quarterly filings—it wasn't (could it be a foreign buyer who did not need to report?)[2] MSN favored a "fat finger" explanation as at least a partial explanation—but the trades in question remained on the tape.[3] The Reddit crowd espoused a variety of alternatives.[4] Stock buybacks by Berkshire—the data did not bear this out. Those wily hedge funds—it turns out filings data showed they were selling more than they were buying. Even the divorce of Bill and Melinda Gates was to blame for the extra volume—did they have to sell the stock to divide up their assets? (turned out the Gates foundation owned BRK.B not BRK.A). Still, volume remained high—and continues so to this day.[5]

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[2] See "Berkshire Hathaway's Mystery Buyer could be revealed soon," at https://global.factiva.com/hp/printsavews.aspx?pp=Print&hc=Publication and https://www.barrons.com/articles/surge-in-berkshire-hathaway-trading-eases-the-mystery-remains-51618244234

[3] See https://www.msn.com/en-us/money/topstocks/warren-buffett-s-berkshire-hathaway-class-a-shares-briefly-soared-51-to-661-504-on-a-single-trade/ar-AAQ0CKQ

[4] https://www.reddit.com/r/BerkshireHathaway/comments/nrggq3/brka_stock_volume/

[5] On July 8, 2022 BRK.A closed at a price of $421,800 on volume of 2,008 shares.

This paper offers a solution to this mystery. We argue that the answer lies in the confluence of three factors: a well-intentioned but ultimately distortionary FINRA rule, trade reporting practices by two retail brokerage firms (initially, RobinHood Securities and later Drivewealth LLC), and fractional shares. As we discuss, the basic problem is that the tape is wrong—and we believe it is likely that many, or maybe even most, market participants had no idea that this was the case. We provide strong evidence that FINRA reporting rules required RobinHood and Drivewealth (two of the largest brokers for fractional shares) to report each fractional trade for BRK.A—even if backed by only a dollar of real money—as a one share trade of a roughly $500,000 stock. Since virtually all trades in BRK.A are one share, when Robinhood commenced reporting fractional trades to the tape in early 2021, the result was a mechanical surge in notional trading volume for BRK.A. A similar surge occurred in October 2021 when Drivewealth commenced reporting fractional trades. We document both such surges below.

While solving the mystery of the surging volume of BRK.A is one contribution of this paper, our real focus lies with bigger issues. First, in an efficient market, how could such a minor change induce large effects? As we show, the FINRA rule we study has been publicly disclosed for years, and Robinhood and other retail brokerage firms have been abundantly transparent about their fractional trade offerings—indeed, they often tout the ability to trade just a fraction of BRK.A. Given these facts, could the market for BRK.A truly be fooled into believing that trades representing perhaps just a few hundred dollars represented *billions* of dollars of trading interest? Or was the confusion limited to Reddit commentators and other "noise" traders?

To explore this issue, we pursue two approaches for investigating price distortions of publicly-traded equity securities. The first exploits the inter-connected nature of BRK.A and BRK.B to examine whether the surge in BRK.A's trading volume was associated with a premium in the price of BRK.A relative to BRK.B. In the process, we develop a new methodology for investigating pairs trading with asymmetric conversion rights. Unlike more standard pairs trading, the ability to convert between shares of BRK.A and BRK.B is asymmetric: a holder of a share of BRK.A has the right to convert into 1,500 shares of BRK.B, but a holder of BRK.B has no conversion right into BRK.A. We model these types of security pairs by combining two traditional time-series econometrics techniques: cointegration and threshold autoregression. Using this approach, we demonstrate how the commencement of

fractional share reporting in BRK.A by Robinhood was associated with a stunning arbitrage opportunity, albeit one that a confused market seemed unable to exploit.

The second approach to examining how the change in reporting of fractional trading led to distortions in the market focuses on intraday liquidity. Robinhood's sudden change in reporting practices to comply with FINRA's rule led to the misleading appearance of a surge in demand for BRK.A relative to that of BRK.B. Using data from the NYSE Trade and Quote (TAQ) database as well as order book data from MayStreet, we present evidence indicating that, while at least some market participants may have been confused about the sudden surge in BRK.A's trading volume, the market makers and specialists who posted orders at the top of the order book may have understood the surge in volume for what it was. In particular, we find no evidence of any change in the expected five-minute price impact for trades in BRK.A during our sample period, consistent with market makers and specialists viewing the new BRK.A trades as being uninformed.

At the same time, however, we also find evidence that quoted and effective spreads for both BRK.A and BRK.B widened following the Robinhood's commencement of fractional trade reporting, indicative that at least some market makers may have widened spreads in response to uncertainty about the sudden increase in BRK.A's trading volume. Additional evidence of heterogeneous effects among liquidity providers also appears in our examination of the full order book for BRK.A. Specifically, we find that, while the most aggressive liquidity providers may have understood the reason for BRK.A's enhanced trading volume in early 2021, and reacted only minimally to the informational shock, other liquidity providers promptly exited the market after fractional trades first began distorting the tape.

Finally, a third contribution is to identify how a well-meaning rule change could have such unforeseen consequences. As we show, the FINRA reporting rule for fractional trading has created significant distortions in the consolidated tape, and as our pairs trading and liquidity analyses reveal, these distortions likely shaped the trading behavior of at least some market participants. In this fashion, our evidence illustrates the capacity of FINRA's fractional trade reporting rule to distort the proper functioning of U.S. equity markets, particularly for high-priced stocks such as BKR.A. Indeed, given its unusually high stock price, BRK.A is in many ways simply an exaggerated example of how FINRA's rule along with the rise in fractional trading creates phantom, non-existent trading volume across all stocks. In this regard, it is

perhaps unsurprising that the FINRA rule we assess caught many brokerage firms such as Robinhood unprepared and flatfooted, while investors and other market observers struggled to make sense of the surge in trading volume. We propose some alternative solutions to the issue of how to deal with fractional shares in market data.

Our paper is related to several recent papers on retail trading. Boehmer, Jones, Zhang, and Zin (2022) propose a metric to identify retail trades and find that positive retail trade imbalance predicts price movements. Welch (2021) investigates Robinhood trading and its impacts, finding that Robinhood traders generally tilted towards high volume, high priced stocks with results suggesting both good timing and good alpha. Barber, Huang, Odean and Schwarz (2021) find that Robinhood traders engage in more attention-induced trading than other retail traders. To our knowledge, the only other paper looking at fractional share trading is Da, Fang, and Lin (2022). These authors do not have data on actual fractional trades but provide an interesting event study of the market impacts when four retail brokerage firms first introduce the ability to trade fractional shares. They find that fractional trading generates price pressures and reversals for high priced stocks during attention generating events. Lastly, from a methodological perspective, our work is also related to the literature on pairs trading (see, for example, Gatev, Goetzman and Rowenhorst (2006)). The model we develop for trading where conversion rights are asymmetric is applicable to many convertible securities, including convertible common and preferred stock, as well as convertible bonds.

The remainder of this paper is organized as follows. Section 2 sets out the issues connected with BRK.A trading and relates these to the general issue of fractional share trading. We also discuss FINRA reporting rules and how the efforts of RobinHood and Drivewealth—the retail brokerage firms with sizeable fractional trading in BRK.A—to comply with the reporting rule resulted in a sharp increase in BRK.A trading volume at two points in 2021. Section 3 then investigates the trading of BRK.B and BRK.A, with a particular focus on the cointegration of their prices and the best way to incorporate asymmetries in the conversion rules. Section 3 concludes by showing how the commencement of fractional share reporting by Robinhood in early 2021 resulted in an arbitrage opportunity for traders. Section 4 turns to our intraday analysis of how fractional trade reporting distorted the provision of liquidity in both BRK.A and BRK.B. Section 5 discusses the FINRA rule and fractional trades, considers what is the

appropriate way to address both transparency and accuracy for the tape, and proposes some reporting alternatives. Section 6 concludes.

2. Trade Reporting Rules for Fractional Trading in BRK.A

A. Fractional Trades and the "Rounding Up" Rule

In general, retail broker-dealers use two different methods for executing fractional trades for their customers. The first method relies on the broker-dealer's clearing firm to execute the transaction, with the clearing firm bearing the market risk with respect to the "excess" shares. For instance, many brokerage firms such as SoFi, Betterment, M1 and Stash, among others, clear using Apex Clearing, which has facilitated fractional trading since late 2018.[6] To illustrate, if SoFi seeks to execute a fractional buy for 3.2 shares of an issuer on behalf of a customer, Apex will execute this transaction by "rounding up" and purchasing four shares of the stock in the market, allocating 3.2 shares to SoFi's account and 0.8 shares to Apex's "fractional inventory account" that it manages through its proprietary trading desk.[7] As a result, the trade will appear in the consolidated tape as a trade for four shares, which accurately reflects the dollar value of trading that occurred because of this transaction.

In contrast, other retail brokerage firms execute fractional trades against their own inventory. This is analogous to the same way that a retail market maker might internalize a whole share trade. While Robinhood previously cleared through Apex, beginning in late 2018 it switched to self-clearing.[8] One consequence of its transition to self-clearing is that fractional trades at Robinhood are today executed against its own inventory, as clarified in its customer agreement:

> *You understand that when Robinhood executes orders that include a Fractional Share ("Fractional Orders") utilizing inventory held in its principal account, the*

[6] See https://www.businesswire.com/news/home/20181023005447/en/, last accessed April 28, 2022.
[7] Additional details concerning Apex's method for executing fractional trades are provided in the registration statement relating to its (aborted) 2021 merger with Northern Star Investment Corp. II. See Northern Star Investment Corp. II, Amendment No. 2 to Form S-4 (filed May 24, 2021). As noted there, "Apex Fintech does not execute fractional share orders against its own inventory. When executing customer orders for fractional shares, after validating the order, Apex Fintech rounds the orders to the next whole share and sends a market (or limit, depending on the customer order received) order to the market. When the order is filled, the shares received are placed into Apex Fintech's fractional inventory account, whereby Apex Fintech then allocates the fractional shares to the customer's account and moves the residual, or otherwise unallocated fractional share, to Apex Fintech's own inventory account. Apex Fintech's inventory account is managed by the Apex Fintech trading desk. Typically, when the whole share quantities exceed internal quantity or notional thresholds, Apex Fintech reduces its positions to ensure Apex Fintech does not carry excessive risk."
[8] See https://www.cnbc.com/2018/10/10/robinhood-launches-its-own-trade-clearing-system-as-customer-growth-surges.html, last accessed April 28, 2022.

portions of such Fractional Orders that execute against inventory are executed in a principal capacity.[9]

This method of fractional trading is also utilized by Fidelity, Charles Schwab, Interactive Brokers, as well as the back-end brokerage firm Drivewealth.[10] Through its customizable suite of APIs, Drivewealth is the brokerage firm behind "micro-investing" platforms such Revolut and Cash-App that allow their customers to trade in fractional shares and/or invest their "spare change" in fractional shares.[11]

Critically, because a firm using this latter method executes a fractional trade against its own inventory, each fractional trade must be reported separately to a FINRA trade reporting facility, just as each trade internalized by a retail market maker must be reported to a FINRA trade reporting facility.[12] The primary difference is that each trade is for less than a single share, raising the question of how the retail brokerage firm should report the share quantity. The obvious answer might be the relevant fraction, perhaps rounded to some number of decimals, or both the dollar value of investment (the "numerator" of the fraction) and the price of the stock (the "denominator" of the fraction).

However, in fact the answer, as provided by FINRA since 2017 in its "Trade Reporting Frequently Asked Questions," is to "round up" any fractional transaction to the nearest whole share.[13] Because of this rule, a trade for one-half, one-third, one-hundredth—or even less!—of a

[9] See Robinhood Financial LLC & Robinhood Securities, LLC Customer Agreement (April 22, 2022), available at https://cdn.robinhood.com/assets/robinhood/legal/Robinhood-Customer-Agreement.pdf.

[10] See, e.g., Fidelity, https://www.fidelity.com/trading/fractional-shares (When processing fractional and dollar-based orders, Fidelity Brokerage Services (FBS) will act as agent and National Financial Services (NFS) will act in a mixed capacity (as principal for the fractional share components and as agent for the whole share components) when executing an order."); Drivewealth, https://legal.drivewealth.com/fractional-shares-disclosure ("When executing on a Principal Basis, DriveWealth will execute the fractional component of the order against its principal facilitation account").

[11] For instance, Revolut will "round up" credit card transactions to the nearest whole increment, placing the difference between the rounded-up number and the actual credit card charge into a savings account that can be invested in fractional shares. Formally, platforms such as Revolut and Cash-App serve as "introducing brokers" and DriveWealth serves as the broker that executes and clears trades.

[12] The SEC has required since March 2007 that all off-exchange transactions be reported to a formal FINRA-managed Trade Reporting Facility. As described by O'Hara and Ye (2011), this requirement means that off-exchange trades made through a broker-dealer internalizer or in a dark pool (both of which were historically reported to an exchange and then consolidated with the exchanges' own trades when reported) are now effectively segregated and reported to the appropriate SIP as having been executed at a FINRA TRF.

[13] See FINRA, Trade Reporting Frequently Asked Questions No. 101.14, available at https://www.finra.org/filing-reporting/market-transparency-reporting/trade-reporting-faq ("When reporting a trade for a fractional number of shares, firms should delete the fraction and report the whole number, except if the whole number would be 0 (zero). If the whole number would be 0, firms should round up to 1."). Question 101.15 specifically askes "Must trades for less than one share be reported?", to which FINRA responds: "Yes. As noted in FAQ 101.14, where a trade is executed for less than one share, e.g., 1/3 share, firms should round up and report a share quantity of 1."

share will be reported to FINRA, and thus reported on the consolidated tape, as a trade for one whole share.

B. Rounding Up Fractional Trades at Robinhood and Drivewealth.

Given this "Rounding Up" rule, the commencement of fractional trading by retail brokerage firms who adopted the second method of fractional trading obligated them to report each fractional trade to FINRA as constituting a whole share trade in the relevant issuer. This rule has been especially consequential for Robinhood and Drivewealth given the large number of fractional trades these firms execute, as well as their commitment to allowing customers to trade in BRK.A for as little as $1.00 per trade—a stock that many other brokerage firms (e.g., SoFi) expressly exclude from their fractional trade programs. For reasons that are not entirely clear, and perhaps simply reflecting a lack of awareness of the rule, neither Robinhood nor Drivewealth initially complied with this reporting obligation when they commenced their fractional trade programs. Indeed, as a publicly-traded firm, Robinhood expressly acknowledged this reporting deficiency—as well as its efforts to rectify it, upon being notified by FINRA of its obligations—in its 2021 annual report on Form 10-K.[14] As a result, the volume for BRK.A reveals two discontinuous increases in 2021 when these two firms commenced complying with the "rounding up" reporting requirement (Figure 2).

[Figure 2 about here]

To illustrate the impact of these firm's fractional trades on the reported trades in BRK.A we turn to the daily trade files in the NYSE Trade and Quotation (TAQ) Data. These data reflect all trades reported to the consolidated tape and include, in addition to a trade's size and price, information such as whether the trade was executed on an exchange or off-exchange by a FINRA member, the time the venue executed the trade (timestamped in microseconds), the time the trade report was processed by one of the two Securities Information Processors responsible for aggregating trade reports (timestamped in microseconds) and, for off-exchange trades, whether the FINRA trade reporting facility at Nasdaq or NYSE received the initial trade report.

[14] See Robinhood Markets, Inc. Form 10-K, at p. 41. ("[W]hen we launched our fractional shares program in late 2019, based on our understanding of the reporting requirements we did not report proprietary fractional trades to FINRA's Trade Reporting Facility. Since then, FINRA has informed us that such trades should be reported. As a result, we began reporting fractional shares in January 2021, and we continue to work with FINRA to complete back reporting, which may result in fines or penalties for failing to do so at the time of the trades.")

Given these data, Bartlett, McCrary and O'Hara (2022b) find that fractional trades executed by Robinhood and Drivewealth possess a unique "signature" within the TAQ trade data based in part on their distinctive trade reporting latencies. Based on these proxies for Robinhood and Drivewealth fractional trades, Figure 2 estimates the total number reported by Robinhood and Drivewealth to the consolidated tape for BRK.A between January 1, 2020 and April 29, 2022. As shown in the figure, virtually no trades in BRK.A reflected trading by either firm until Friday, February 12, 2021, when approximately 150 trades having the Robinhood "signature" appeared in the tape.[15] These trades increased quickly enough that the following Friday, February 19, 2021, saw over 1,000 trades in BRK.A reflecting the Robinhood signature. Since that time, the number of Robinhood signature trades has fluctuated between 1,000 to 2,000 per day. Broadly, the patterns for Drivewealth are similar, but Drivewealth got a later start. The tape reveals virtually no Drivewealth fractional trades until 163 trades with Drivewealth's "signature" appeared on October 7, 2021. After that, the number of such trades increased to roughly 500 trades per day for the remainder of 2021 and in 2022 there were routinely more than 1,000 such trades.

To illustrate the influence of these trades on BRK.A's total number of trades and trading volume, Figure 3 plots the total number of all fractional trades in BRK.A reported to the tape during the same timeframe (left y-axis), as well as overall trading volume (right y-axis). To facilitate comparison with Figure 1, two vertical lines have been imposed for February 16, 2021 and October 7, 2021, corresponding to the onset of public tape reporting of fractional trades for Robinhood and Drivewealth, respectively. Figure 3 highlights how the sudden surge in fractional trades and trading volume in BRK.A in February 2021 coincided with the exact days on which Robinhood commenced reporting trades to the consolidated tape. While Drivewealth's reporting to the public tape was less of an episodic shift than Robinhood's, the strong trend in Drivewealth's fractional trades documented in Figure 2 leads to a strong trend in fractional trades through April 2022. Figure 3 makes it clear that this trend is mirrored in BRK.A's trading volume. A simple bivariate regression of fractional trades on trading volume over this period has

[15] This is the same week that Robinhood CEO Vladimir Tenev testified (virtually) in the House of Representatives. See his written testimony of February 18, 2021 at https://docs.house.gov/meetings/BA/BA00/20210218/111207/HHRG-117-BA00-Wstate-TenevV-20210218.pdf.

an R^2 of over 0.95, meaning that fractional trades explain virtually all of the variation in trading volume over this time period.[16]

[Figure 3 about here]

The sudden appearance of Robinhood and Drivewealth trades in February and October 2021 naturally raises the question of how much BRK.A's trading volume might have been distorted by the "rounding up" rule that applied to these fractional trades. During this time period, the price of BRK.A ranged from $340,000 to $540,000, yet fractional trades in BRK.A could be placed using the Robinhood and Drivewealth platforms for as little as $1.00 per trade. Because customer data on these fractional trades is not available, there is no direct way to estimate this overage.

We can, however, impute a very rough estimate using information on customer account size. In particular, in Congressional testimony in February 2021, Robinhood reported that the median account value for its customers is approximately $240.[17] Suppose we assume that each trade with the Robinhood or Drivewealth signature was backed by just $240 of actual cash—an admittedly Herculian assumption as this would imply that the median customer's account held only this one trade and so could dramatically underestimate the true reporting average. But if this were the case, then every Robinhood or Drivewealth fractional trade is inflated by the "round up" rule by price less $240. For each day, we aggregate the amount of inflation using this measure, and then normalize by the aggregate trading volume for BRK.A. Figure 4 plots this ratio over time. The figure shows that after Robinhood began reporting its trades to the tape, per FINRA's request, roughly 80 percent or more of BRK.A trading volume has been fictitious.

[Figure 4 about here]

We next tackle the question of whether the sudden surge in trades reported by Robinhood and Drivewealth was occurring at the same time as other changes in the market. For example, it is conceivable that there was an increase in trading in Berkshire Hathaway generally around the same time that Robinhood and Drivewealth began reporting fractional trades to the tape. Figure 5 examines trading volume for BRK.B (panels A and B) and BRK.A (panels C and D) separately

[16] This regression has an estimated slope coefficient of 0.509. This means that an increase of 500 fractional trades—which one could argue means an aggregate investment of less than $1,000—would be predictive of an increase of $255 million in trading volume.

[17] Based on congressional testimony of Robinhood CEO Vladimir Tenev. See https://docs.house.gov/meetings/BA/BA00/20210218/111207/HHRG-117-BA00-Wstate-TenevV-20210218.pdf.

for off-exchange (FINRA) trades and on-exchange trades. Examining first the patterns for BRK.B, we see that generally speaking, exchange trades are twice as important as FINRA trades. For example, for BRK.B, peak trading volume for both FINRA and exchange trades is on March 12, 2020, when there are over twice as many exchange trades as FINRA trades. On average from 2019 to 2022, there is almost exactly twice as much trading volume on- as opposed to off-exchange. For BRK.B, neither FINRA nor exchange trades exhibit any pronounced shift in trading volume around mid-February 2021. In particular, comparing trading volume around February 16, 2021, to trading volume around March 13, 2020, it seems as though trading interest in BRK.B was no higher on February 16, 2021, than it was on March 13, 2020. Turning to BRK.A, we see that trading volume in exchange trades looks similar to that observed for BRK.B—there is no pronounced shift in trading interest around February 16, 2021 (except for a one-day spike that is plausibly a reaction to the surge in FINRA trades). For example, looking only at exchange trades, trading interest in BRK.A seems somewhat lower around February 16, 2021, than it was on March 13, 2020. The pattern for FINRA trades shows that the surge in trading volume in BRK.A around February 16, 2021, documented in Figure 1 is driven virtually entirely by the surge in FINRA trading volume. This analysis is inconsistent with there being a significant factor in the market aside from the sudden reporting of fractional trades by Robinhood and Drivewealth.

[Figure 5 about here]

It is conceivable that the increase in Robinhood and Drivewealth reporting coincides with an exogenous shock in retail trading, i.e., something else that would drive FINRA trading volume. While such a coincidence would seem improbable, we believe we can rule it out empirically. This is possible because in 2021 both Robinhood and Drivewealth also commenced a process of retroactively correcting their trade reporting deficiencies to a separate FINRA reporting program. This allows us to observe the number of fractional trades that occurred at these firms prior to the date they began reporting them to the tape.[18]

The FINRA reporting program alluded to above, the "OTC Transparency" initiative, was designed to provide greater insight into trading volume that occurs off exchanges in either an

[18] As indicated in note 14, Robinhood expressly noted that it was working with FINRA to "complete back reporting."

Alternative Trading Systems (ATS) or by a retail market maker. Since 2016, FINRA has published on its website weekly summaries of the total number of trades and shares executed by FINRA member firms on a stock-by-stock basis for trades on both ATS and trades that are internalized.[19] In other words, the OTC Transparency data reveals for each issuer which trading firms account for trades in the issuer that appear in the consolidated tape as having been executed off-exchange. Moreover, while the public tape is never retroactively corrected, the OTC Transparency data are—and these data show significant activity by Robinhood and Drivewealth that match our proxy almost exactly, as we next show.[20]

To ensure comparability with the OTC Transparency data, which report weekly as noted, we aggregate our proxies for Robinhood and Drivewealth from Figure 2 into weekly data as well. Panel A of Figure 6 compares the back-filled trades disclosed by Robinhood to FINRA (represented by solid bars) with the weekly estimates for Robinhood based on the trades in the tape having the Robinhood "signature" (represented by dots). As shown by the solid bars, fractional trades in BRK.A at Robinhood first appear in the FINRA OTC Transparency data for the week of August 31, 2021 and averaged roughly 5,000 trades per week until the end of January 2021, when they increased to roughly 10,000 trades per week. Thereafter, fractional trading in BRK.A at Robinhood remained at this level through March 2021, thus revealing that the large number of trades appearing in the tape in February 2021 was not due to a sudden surge in fractional trading in BRK.A among Robinhood's customers.

[Figure 6 about here]

Panel B of Figure 6 provides an analogous analysis for Drivewealth trades. Note that, in contrast to Panel A, FINRA presently lacks weekly trades for Drivewealth from August 9, 2021 through October 11, 2021. These missing data reflect the fact that, as of when we pulled the OTC Transparency data, Drivewealth had not yet completed the back-filling of its trade reports for FINRA.[21] (The FINRA data indicate that the most recent update to Drivewealth's FINRA data was February 17, 2022). Nevertheless, the figure reveals that between January 2021 and June

[19] See FINRA Regulatory Notice 15-48, Equity Trading Initiatives: OTC Equity Trading Volume, available at https://www.finra.org/sites/default/files/Regulatory-Notice-15-48.pdf

[20] Robinhood's backfilling of their FINRA data was not without controversy. For instance, on Reddit, the fact that Robinhood was backfilling the data for OTC Transparency initiative fed directly into various conspiracy theories concerning trading in GME in 2021. See, e.g., https://www.reddit.com/r/Superstonk/comments/p4w9hq/january_gme_otc_trades_increased_by_32_last_week/.

[21] We last pulled the OTC data in March 2022.

2021, Drivewealth completed roughly 2,000 fractional trades in BRK.A per week, which is roughly the amount of trades executed by Drivewealth after it commenced reporting trades to the tape in October 2021.

More generally, in both panels of Figure 6, comparison of the weekly trade reports disclosed by FINRA with the weekly proxies based on their trade "signatures" underscores the utility of using these proxies to identify Robinhood and Drivewealth trades within TAQ's intraday trade data. In the case of Robinhood, this proxy for Robinhood trades captured nearly 90% of the weekly trades reported by Robinhood to FINRA. For Drivewealth, the accuracy rate was over 99%; indeed, for the final eleven weeks of 2021, the difference between the weekly shares disclosed to FINRA and the weekly number of proxied shares was never more than 5 shares, with 3 weeks matching perfectly.

3. Did the Distorted Trading Volume in BRK.A Distort its Prices?

Given the findings from Section 2, a natural question to ask is whether the inflated trading volume since February 2021 was associated with other distortions in the market for Berkshire Hathaway's common equity. In this Section, we focus on distortions in the trading price of BRK.A relative to BRK.B. In Section 4, we turn to distortions in intraday liquidity.

As noted in the Introduction, journalists and Reddit commentators alike offered no shortage of theories for the spike in trading volume in BRK.A in early 2021, and many of these theories might be expected to affect the price of BRK.A relative to BRK.B. For instance, as we describe below, a distinguishing feature of BRK.A is the fact that each share possesses superior voting rights relative to a share of BRK.B. If traders believed that a "mystery buyer" was building a position in BRK.A but not BRK.B, it would be reasonable for them to infer that it was because of these superior voting rights. Moreover, given the unique voting *and conversion* features of BRK.A, shares of BRK.A should trade at a premium to those of BRK.B.

These voting and conversion features of BRK.A date back to 1996 when Berkshire Hathaway first issued shares of its Class B common stock.[22] At inception, a B share had dividend and distribution rights equal to 1/30th those of an A share, on the one hand, and voting rights equal to

[22] Class B shares were first issued on May 8, 1996 and Berkshire Hathaway's existing common stock was renamed Class A shares. See https://www.berkshirehathaway.com/1996ar/common.html.

1/200th those of an A share, on the other.[23] Their relationship was asymmetric not just in terms of voting, but also in terms of conversion rights: A shares had the option of converting into 30 shares, but B retained no such conversion rights. While the A shares, somewhat famously, have never split, the B shares underwent a 50-for-1 split in early 2010.[24] With that split, the 1/30th and 1/200th economic and voting rights were amended to 1/1,500th and 1/10,000th, respectively, and A's 30:1 conversion rights were similarly amended to 1,500:1. These ratios hold today. Zooming out for the big picture, one might say that today shares of BRK.A are wildly expensive voting shares and those of BRK.B are normal economic shares.

Because BRK.B is so much more affordable than BRK.A, and because the two securities are expected to track each other so closely, it is not surprising that BRK.B is many times more liquid than BRK.A.[25] Figure 7 shows the total number of shares traded for the two securities over our study period. The total shares for BRK.A are displayed on the left axis in thousands, while that for BRK.B are displayed on the right axis in millions. Focusing on a date before the distortionary effect of fractional trades documented above, in particular the highly active date of March 13, 2020, we see that BRK.A had an active day, with 1,075 shares traded. In contrast, on that same date, BRK.B saw 14,867,020 shares traded. Generally, throughout 2020 and before the advent of fractional trading on the tape, a typical day for BRK.A saw just under 500 shares trade hands, but for BRK.B the analogous figure is over 6 million, or 14,000 times greater. Clearly, then, we should expect that there will be much more efficient price discovery for BRK.B than for BRK.A.

[Figure 7 about here]

These considerations lead naturally into a cointegration model, where we use the price of BRK.B to predict the price of BRK.A. Taking into account the scale differences discussed above in the period after the stock split in 2010, we posit a multiplicative model for the price level of BRK.A, A_t, and the price level of BRK.B, B_t:

$$A_t = 1500 B_t \varepsilon_t \tag{1}$$

[23] For economic rights at inception, see the footnotes to https://www.berkshirehathaway.com/1996ar/consolid.html, and for voting rights at inception, see note (11) to "Notes to Consolidated Financial Statements" at https://www.berkshirehathaway.com/1998ar/1998ar.pdf.
[24] See https://www.berkshirehathaway.com/2009ar/2009ar.pdf.
[25] Practically, BRK.B functions in some ways as an "ETF" for BRK.A.

where ε_t is independent of B_t and has unit expected mean. Then in logs we have

$$a_t \equiv \ln A_t = \ln 1500 + \ln B_t + \ln \varepsilon_t \equiv \ln 1500 + b_t + e_t \tag{2}$$

where a_t and b_t are log prices and e_t is independent of b_t with a negative mean.[26] Since they are log prices, a_t and b_t are expected to be approximately random walks, especially over time grids that are not particularly fine.

If a_t and b_t are in fact random walks and if, in contrast, e_t is ergodic, then a_t and b_t are said to be cointegrated. There is ample evidence along these lines, as we next summarize. In particular, using either the Dickey-Fuller or Phillips-Perron test of the null hypothesis that the series contains a unit root (where the alternative is that the series is ergodic) leads to little evidence against the natural null that a_t and b_t contain a unit root, but decisive evidence against that same null for e_t.

To describe these results, let us start by noting that there are two obvious ways to measure daily prices for BRK.A and BRK.B. One approach is to use closing prices from the Center for Research in Securities Prices (CRSP). The other is to use size-weighted average transaction prices (VWAP) from the TAQ data. Table 1 summarizes these unit root tests for either measurement approach using all trading days between January 2, 2020 and July 1, 2022.

[Table 1 about here]

For log BRK.A (a_t) and log BRK.B (b_t), measured either using closing prices or intraday VWAP, Dickey-Fuller test statistics (with no drift or trend terms) are comfortably to the right of even the 10th percentile of the test statistic distribution under the null of there being a unit root. This provides little evidence against the natural null hypothesis that is in fact rather natural, namely that the log of a stock price has a unit root. To operationalize the residual e_t, we regress a_t on b_t and use the fitted residuals $\hat{e}_t$ in place of e_t to test for a unit root.[27] Doing so yields a Dickey-Fuller test statistic far to the left of even the 1st percentile of the null distribution. Results for either of the two Phillips-Perron test statistics is similar: while a_t and b_t both have

[26] While one might say that that $E[e_t] \approx 0$, more precisely Equation (1) implies that $\mathrm{E}[\ln A_t - \ln B_t] = \ln 1500 + E[\ln \varepsilon_t]$. Jensen's inequality means that $E[e_t] = E[\ln(\varepsilon_t)] \leq \ln(E[\varepsilon_t]) = 0$.

[27] A key result from time series econometrics, formalized by Theorem 2 of Engle and Granger (1987, see also discussion pp. 263-64), is that parameter estimates from the regression implied by equation (2) are T-consistent. This means we are justified, asymptotically as $T \to \infty$, in ignoring the estimation error and treating the estimated residual in equation (2) as if it were in fact e_t. This is often referred to as the Engle-Granger approach to cointegration (as opposed to the Johansen (1991) maximum likelihood method, which we do not pursue here).

Phillips-Perron statistics that are consistent with the series having a unit root, we decisively reject the null of a unit root for the fitted residuals $\hat{e}_t$.

By virtue of the wide variation in a series with a unit root, the regressions just described—of log BRK.A on log BRK.B—is itself a T-consistent approach to estimating the cointegrating relationship. Table 2 shows the results of regressions along those lines, both for closing prices and for intraday VWAP using all trading days between January 2, 2020 and July 1, 2022. The first column corresponds to intraday VWAP and the second column corresponds to closing prices. The point estimate indicates that the model in Equation (2) is highly accurate. The constant term is estimated to be either 7.28 or 7.29, with an upper bond of the 95% confidence interval being right around 7.30.[28] The other implication of Equation (2), namely that the coefficient on b_t is one, is also corroborated in data. For both intraday VWAP and closing price, the point estimate for the slope term differs from one in only the third decimal place (1.005 and 1.004, respectively).

Finally, the cointegrating relationship is estimated with tremendous precision almost regardless of the approach taken to variance estimation. Heteroskedasticity-consistent standard errors are on the order of 0.005 or less, and heteroskedasticity-and-autocorrelation-consistent (HAC) standard errors are on the order of 0.01 or less. Generally speaking, our estimated HAC standard errors are slightly more than twice as large as their heteroskedasticity consistent cousins. While not displayed, the R^2 from these regressions exceeds 0.999 in each instance.

Taking first differences of the model in Equation (2), we obtain a model for the returns for BRK.A as a function of the returns for BRK.B:

$$\Delta a_t = \Delta b_t + \Delta e_t \tag{3}$$

That is, the constant of $\ln 1500$ differences out and it remains an implication that the coefficient on the returns to BRK.B is one.

Moreover, thinking of the logic of Equation (2), note that in any period when e_t is high, the price of BRK.A is too high relative to the benchmark of 1500 times that of BRK.B—and we would expect for the price of BRK.A to fall in the next period. That is, we would predict a return to equilibrium. This leads to the error-correction model

[28] For reference, $\ln 1500 \doteq 7.313$. The constant term is expected to be somewhat below $\ln 1500$ because of Jensen's inequality, as noted above.

$$\Delta a_t = \Delta b_t + \gamma e_{t-1} + \Delta e_t \quad (4)$$

with the prediction that γ<0. That is, when the price of BRK.A is high relative to equilibrium, we would predict a negative return in the next period as the price comes back down to equilibrium, and when the price of BRK.A is low relative to equilibrium, we would predict a positive return in the next period as the price rises back up to equilibrium.

Results for models for returns for BRK.A according to both the model in Equations (3) and (4) are presented in Table 3.

[Table 3 about here]

Unsurprisingly, the results show that returns to BRK.B are a highly accurate predictor for the returns to BRK.A. The fit of the model is not perfect, but for a model of returns, it is very close. Above, we noted that the R^2 from the cointegrating regression exceeded 0.999. This might not be surprising since it is a model in log levels. However, modeling returns is more challenging than modeling levels—and yet the R^2 from the return regressions all exceed 0.94. While we suppress presentation of the intercept for simplicity of display, it may be of interest that it is estimated to be approximately zero, which is consistent with the model in Equation (2). The most notable defect of these regressions in terms of departure from the theoretical model is that the coefficient on the returns to BRK.B is not exactly one. Moreover, the difference is statistically significant at conventional levels, particularly for the results in panel A, with the 95% confidence interval excluding one.

This suggests that there may be some slight noise involved in both of these proxies for true price. A classical measurement error model would imply that the coefficient on a mis-measured variable would be biased ("attenuated") toward zero. To the extent this is the explanation for the departure of the slope coefficients from one, it is interesting to note that the results in panel B based on the closing prices measure exhibit (1) greater discrepancy from one, and (2) notably larger standard errors, suggesting simply more noise in the measure of closing prices than in intraday VWAP. For this reason, we place somewhat greater stock in our measures that are based on intraday VWAP.

While Equations (3) and (4) might be viewed as classical cointegration results, they do not completely capture a basic asymmetry of the economic relationship between the price of BRK.A and BRK.B. Specifically, and as noted, by virtue of the differential economic and voting rights

of the A and B shares, A should trade at approximately a 1,500 multiple that of B, but the equilibrium forces are asymmetric. As Warren Buffett explained, were the B shares to trade for more than 1/1,500th the A shares, an arbitrage possibility exists, and we would expect for A shareholders to convert to B, rapidly pushing the price ratio back to 1,500. This arbitrage play could be acted upon quickly, within a span of 2-3 trading days, and if many traders did so, then if BRK.A were undervalued relative to BRK.B, we would expect for the price of A to rebound quickly. However, the same is not true if BRK.A were overvalued relative to BRK.B. To quote Buffett: "the B can [for some time] sell for less than 1/1,500th the price of the A since conversion doesn't go in the reverse direction."[29] Indeed, we might particularly expect for A to trade at more than a multiple of 1,500 that of B when the market places a premium on voting rights in Berkshire Hathaway (e.g., as might be expected if the market expects a succession battle).

These considerations lead naturally to an *asymmetric* cointegration model that draws upon the threshold autoregressive (TAR) modeling tradition. In particular, define I_t to be an indicator for whether e_t is positive, i.e., $I_t \equiv 1(e_t \geq 0)$. Then as shown by Petrucelli and Woolford (1984), in the context of the regression model with $\Delta\eta_t$ an idiosyncratic error

$$\Delta e_t = \rho_1 I_{t-1} e_{t-1} + \rho_2 (1 - I_{t-1}) e_{t-1} + \Delta\eta_t$$

there is a trio of necessary and sufficient conditions for e_t to be ergodic: $\rho_1 < 0$, $\rho_2 < 0$, and $(1 + \rho_1)(1 + \rho_2) < 1$. Regression estimates of the parameters ρ_1 and ρ_2 and their associated variance matrix allow for Wald testing of these restrictions; doing so yields evidence against the joint null $\rho_1 = \rho_2 = 0$ and $(1 + \rho_1)(1 + \rho_2) = 1$ and little evidence of the alternative.[30]
These considerations lead into an asymmetric error correction model for returns for BRK.A:

$$\Delta a_t = \Delta b_t + \gamma_1 I_{t-1} e_{t-1} + \gamma_2 (1 - I_{t-1}) e_{t-1} + \Delta\eta_t \quad (5)$$

where, as above, we implement Equation (5) using $\hat{e}_t$ and $\hat{I}_t$ in place of the true e_t and I_t, respectively.[31]

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[29] See https://www.berkshirehathaway.com/brkshareholderinfo/compab.pdf, for discussion.
[30] For example, for intraday VWAP, regression estimates of ρ_1 and ρ_2 are -0.113 ± 0.165 and -0.568 ± 0.208, respectively. Testing the joint restriction that they are both zero yields an F-statistic (2 numerator dof) of over 40 and a p-value well below 0.01. Testing the nonlinear restriction $(1 + \rho_1)(1 + \rho_2) = 1$ leads to a chi-square (1 dof) of over 45 with a p-value well below 0.01.
[31] Note that $\gamma_1 = \rho_1 + \gamma$ and $\gamma_2 = \rho_2 + \gamma$, where γ is from Equation (4). While we do not pursue it here, this suggests a framework for comparing the TAR error-correction model against the standard error-correction model.

In terms of what we expect from the estimated parameters of Equation (5), note again that economically, e_t is a measure of the departure of BRK.A and BRK.B from their equilibrium relationship. When $e_t < 0$, the market places a value on BRK.A that is less than 1,500 times the value is places on BRK.B. Such a situation is not expected to persist for long. As Buffett put it (overly?) sharply, "Class B can *never* sell for anything more than a tiny fraction above 1/1,500$^{\text{th}}$ of the price of A. When it rises above 1/1,500$^{\text{th}}$, arbitrage takes place in which someone… buys the A and converts it into B. This pushes the prices back into a 1:1,500 ratio."[32] This implies that γ_2 will be a major factor in equation (5). In contrast, when $e_t > 0$, the market values BRK.A over 1,500 times that of BRK.B, and this situation—while not the expected situation—can exhibit persistence. This implies that γ_1 will be less of a factor in equation (5). In particular, we expect $|\gamma_2| > |\gamma_1|$.

Estimates of Equation (5) are presented in Table 3 (Model (5)), again using all trading days between January 1, 2020 and July 1, 2022. Particularly for what we believe are the more accurate estimates from panel A, corresponding to intraday VWAP, the predicted asymmetry discussed above is reflected in the estimated parameters. The coefficient on the returns to BRK.B continues to be nearly (but not quite) one and estimated with a great deal of precision. The model here essentially decomposes the speed of adjustment back to equilibrium into a component corresponding to when the disequilibrium is positive (γ_1) and when it is negative (γ_2). As anticipated from the discussion above, the speed of adjustment back to equilibrium is much slower when disequilibrium is positive than when it is negative. When it is positive, the estimated effect is -0.11 or so, whereas it is -0.63 when it is negative. The results for panel B show similar, but much more noisy results. The estimated speed of adjustment when positive is much larger, namely -0.49, but also indistinguishable from the speed when negative, namely -0.60.

We can use the estimated Model (5)—what we view as our best model for the log price of BRK.A—to generate a measure of cumulative abnormal returns. We do so in Model (6), restricting the estimation period to the period of time where we are confident Robinhood had not yet begun reporting fractional trades to the tape, namely the period of time prior to mid-January

[32] Warren Buffett, "Comparative Rights and Relative prices of Berkshire Class A and Class B Stock," https://www.berkshirehathaway.com/brkshareholderinfo/compab.pdf.

2021. Parameter estimates for this model are particularly interesting, because this is also the period of time over which the fractional trade reporting pattern we are studying is not yet in a position to possibly affect the market. The estimated parameters are generally similar, but the speed of adjustment parameters are much more powerful. The positive disequilibrium parameter is estimated to be -0.55 and the negative disequilibrium parameter is estimated to be almost -1. The estimated standard error on the positive disequilibrium is much larger than in Model (5) because it is rare in the period prior to mid-January 2021 to observe large positive deviations. The notable change in the estimated speed of adjustment parameters between Models (5) and (6) is itself indicative that fractional trade reporting has influenced the pricing dynamics of BRK.A.

Using the coefficient estimates from Model (5), we estimate abnormal returns to BRK.A between January 1, 2021 and July 1, 2022 using the fitted residuals from the model for intraday VWAP, and we cumulate them over time to arrive at cumulative abnormal returns. These are plotted in Figure 8, which shows an extraordinary surge beginning around mid-January 2021 and persisting until the present. These daily abnormal returns are remarkable both for their size and persistence—over the year beginning in January 2021 they cumulatively reach 60 percent. How this occurs in a highly visible stock in an efficient market is puzzling indeed. It is conceivably consistent with one or more buyers placing a newly high value on voting power (e.g., the succession battle alluded to above). However, the timing of such a claim is suspicious: the unusual excess demand for BRK.A relative to BRK.B just so happens to coincide with a highly confusing time period for BRK.A, whereby newfound compliance with FINRA's obscure rule leads, as outlined above in Section 1, to a highly distorted public record of transactions. Moreover, the prospect of a succession battle is at least speculative, as Warren Buffett and Charlie Unger have made the path of succession rather clear for some time.[33]

[Figure 8 about here]

In many ways, this model is a formalization of what one can infer from the simple ratio of BRK.A to BRK.B. Figure 9 shows this ratio over time, using the intraday VWAP measure.

[Figure 9 about here]

[33] See, for example, Eric Platt, "Berkshire succession: Greg Abel confirmed as Warren Buffett's heir apparent," *Financial Times*, May 3, 2021.

In summary, BRK.A traded at a premium to BRK.B during our sample period, with the premium arising at roughly the time that BRK.A's volume surged when Robinhood commenced reporting fractional trades in BRK.A. Given the absence of any notable news events that were unique to BRK.A during this time period,[34] we interpret these results as consistent with the possibility that, at the margin, the market price for BRK.A was distorted by the inflated trading volume caused by the reporting of fractional trades.

4. Fractional Share Reporting and Liquidity

We next turn to examining whether similar distortions occurred with regard to the intraday liquidity for Berkshire Hathaway's common equity. As with the trade price for BRK.A, many of the prevailing explanations for the spike in the security's trading volume might also be expected to affect its intraday liquidity. For instance, to the extent the trading volume was due to a large institutional buyer of BRK.A, classical theories of adverse selection (Bagehot, [1971]; Copeland and Galai [1983]) suggest the surge in volume in BRK.A should be accompanied by an increase in the price impact of observed trades and a concomitant decline in the profitability of market-making. For similar reasons, liquidity providers might widen quoted spreads on account of the heightened risk of trading with an informed trader (Glosten & Harris, 1988). More generally, uncertainty itself about what was causing the spike in trading volume in BRK.A could cause liquidity providers to post orders at less aggressive prices for both BRK.A and BRK.B out of concern that better informed traders may be lying in wait.

At the same time, however, there are good reasons to question whether these effects would be observed. Most importantly, virtually all of the increased trading volume in BRK.A was the result of FINRA's "rounding up" rule that applied to the fractional trades executed through Robinhood and Drivewealth. As noted previously, these trades each possessed an identifiable signature that was as viewable to a trader as it is to an academic researcher. Critically, this signature indicated that the trade represented—in all likelihood—a trivial amount of economic trading interest. Moreover, by virtue of their obligations to provide liquidity in BRK.A and BRK.B, market makers and specialists might be especially motivated to uncover the cause of BRK.A's elevated trading volume. For this reason, it is reasonable to assume that repeat playing

[34] As noted below, we conduct a search on Factiva for any news story mentioning Berkshire Hathaway during 2021 to isolate trading days where there was new public information concerning the company or its securities.

market makers and specialists might recognize the surge in volume for what it was: Fractional trades by retail investors that represented little meaningful economic value.

We assess empirically the liquidity effects of the 2021 surge in reported volume for BRK.A in three stages. First, we examine the temporary and permanent price impact for trades in BRK.A surrounding the publication of fractional trades by Robinhood and Drivewealth. Second, we examine the quoting environment for both BRK.A and BRK.B to evaluate how liquidity providers responded to the surge in BRK.A's trading volume during this same time frame. Lastly, using a proprietary dataset for all orders for BRK.A, we examine changes in the order book for BRK.A surrounding the dramatic increase in BRK.A's trading volume in early 2021.

A. Price Impact

Price impact provides a critical liquidity measure for liquidity providers and liquidity takers for separate but related reasons. With regard to liquidity providers, it provides a means to estimate adverse selection costs. In particular, for a given trade at time t and price P, the profitability of the trade for liquidity providers can be estimated by examining the extent to which the midpoint of the NBBO moves in the ensuing five minutes. For instance, for a marketable buy order, a liquidity provider that shorts the stock to fill the order and closes the position at the midpoint of the NBBO in five minutes will earn profits per share equal to the difference between P_t and the midpoint of $NBBO_{t+5}$. For similar reasons, with regard to liquidity takers, price impact provides an estimate for how much a trader should expect prices to move in response to a marketable buy or sell order. All other things being equal, lower measures of price impact indicate the ability to trade large quantities without impacting prices.

A notable challenge in applying this logic after Robinhood and Drivewealth commenced reporting fractional trades to the tape is the need to identify trading direction. This challenge is immediately evident by examining the daily dollar-volume-weighted average effective spread for BRK.A over our sample period, which we show in Figure 10. For this purpose, we define the effective spread of a trade as:

$$\text{Effective Spread} = 2 * |\ln(P_t) - \ln(M_t)|,$$

where P_t is the price of a trade at time t, and M_t is the midpoint of the NBBO at the time of the trade.

[Figure 10 about here]

The notable change in effective spreads after February 16, 2021 and October 7, 2021 reflects the different conventions used by Robinhood and Drivewealth, respectively, for pricing fractional trades. Across the Robinhood and Drivewealth trades that we identify, almost all Robinhood trades were priced at the midpoint of the NBBO, and virtually all Drivewealth trades were priced at either the NBB or the NBO (based on the direction of the order). Moreover, the right y-axis in Figure 10 indicates that reported Robinhood trades represented roughly 80% of the reported dollar volume between February 16, 2021 until Drivewealth commenced reporting trades in October 2021. After that date, Robinhood trades constituted roughly 55% of reported dollar volume, while Drivewealth trades represented roughly 30%.

That such a large fraction of trades was completed at the midpoint of the NBBO complicates estimating liquidity measures, such as price impact, that rely on trade direction. For instance, pursuant to the Lee and Ready convention of assigning trade direction, the direction of a midpoint trade is determined by the "tick test," in which a trade is assumed to be initiated by a marketable buy (sell) order if the most recent prior trade at a different price was at a lower (higher) price than the observed midpoint trade. Within our sample, however, we commonly observe clusters of midpoint trades bearing the Robinhood signature, making it implausible to assume the same trade direction for every trade in the cluster. At the same time, incorrectly assigning trade direction to midpoint trades can bias the calculating of price impact toward zero.[35]

Accordingly, we calculate two measures of price impact to account for the large number of midpoint trades in our sample. Our first measure of price impact, PI_Narrow, follows the conventional approach to estimating the permanent component of effective spreads as follows:

$$\text{PI\_Narrow}_k = 2 * D_k(\ln(M_{t+5}) - \ln(M_t),$$

where D_k is an indicator variable that equals +1 if the kth trade is a buy and −1 if the kth trade is a sell based on whether the trade price P_t is above or below the midpoint price M_t at the time of the trade. However, for purposes of this calculation, all midpoint trades are dropped from the

[35] As an example, consider four buy orders for 1 share completed at $6, $5, $5, and $5, and all four trades are completed when the midpoint of the NBBO is $5. If the midpoint of the NBBO in five minutes is $6, the value-weighted average price impact for these trades would be $1.5. However, using tick test, the three midpoint trades would be assigned sell orders, yielding an estimate for price impact of -$0.50.

dataset before calculating the dollar-volume-weighted average for PI_Narrow. Our second measure of price impact, PI_Broad, is calculated as:

$$PI\_Broad_k = 2 * |\ln(M_{t+5}) - \ln(M_t)|,$$

which is calculated for all trades, including midpoint trades. By estimating the absolute value of the log-difference between M_{t+5} and M_t, PI_Broad assumes that any change in M_{t+5} relative to M_t is consistent with the trade's direction. This measure is thus biased in favor of detecting price impact.

We plot the daily dollar-volume-weighted average for PI_Narrow and PI_Broad in Figure 11. Turning first to PI_Narrow, note first that while this measure excludes the price impact arising from Robinhood's midpoint trades, it nevertheless captures the price impact of the large number of Drivewealth trades occurring after October 7, 2021, which is highlighted by the second vertical, red line in the graph. Notably, the estimated price impact for BRK.A turns largely negative after this date, indicating that the likelihood that M_{t+5} would decrease (increased) following a buy (sell) trade by a Drivewealth customer—the opposite of what would be expected if market participants were inferring that these trades were informed. Likewise with respect to PI_Broad, Figure 11 provides very little evidence of any material change in this measure between February 16, 2021 and October 7, 2021, the period when Robinhood trades represented over 90% of the trading volume in BRK.A.[36] After October 7, 2021, the measure ticks up gradually, which should be expected given that PI_Broad includes the absolute value of PI_Narrow for the Drivewealth trades reported after this time.[37]

[Figure 11 about here]

[36] We attribute the modest spike in PI_Broad shortly after February 16, 2021 to volatility surrounding the release of Berkshire's Q4 2020 financials, as we discuss in the following subsection.

[37] In unreported results, we also estimate two versions of daily realized spreads using a similar approach. Specifically, we calculate RS_Narrow as $2 * D_k(\ln(P_t) - \ln(M_{t+5})$ as well as RS_Broad as $2 * |(\ln(P_t) - \ln(M_{t+5})|$. While price impact is generally understood as an estimate of the adverse selection costs faced by market makers, realized spreads can be viewed as an estimate for the temporary component of the effective spreads and therefore the profitability of providing liquidity in a security. In other words, in absence of price impact, realized spreads would equal the effective spread because that is the profit a market maker would receive from providing liquidity and closing out the trade. As one would expect from Figure 11, our estimates reveal no notable change in realized spreads between February 16, 2021 and October 7, 2021. After this latter date, however, both estimates for realized spreads increase notably, consistent with the fact that (a) Drivewealth executed trades at the NBB or NBO (thus ensuring that the effective spread for these trades is equal to the quoted half-spread) and (b) there is very little price impact after this date.

Lastly, we complement this high-frequency analysis of price impact, by examining changes in daily Amihud illiquidity, a common low-frequency proxy for price impact (Goyenko et al., 2009). Specifically, Amihud (2002) develops a price impact measure that utilizes daily return and volume data to estimate the daily price response associated with one dollar of trading volume. This "illiquidity" measure is defined as:

$$Illiquidity = Average\left(\frac{|r_t|}{Volume_t}\right)$$

where r_t is the stock return on day t and $Volume_t$ is the dollar volume on day t. The formulation of this measure is particularly relevant for examining the price impact of the sudden increase in trading volume in BRK.A after February 16, 2021. In particular, if prices were unresponsive to the surge in trading volume after this date, Amihud Illiquidity should drop precipitously with the spike in reported trading volume. As shown in Figure 12, this is precisely what we find.

[Figure 12 about here]

Overall, our price impact analysis suggests that the surge in BRK.A's trading volume that occurred when Robinhoood and Drivewealth began reporting fractional trades—and the associated rumors of an informed buyer—did not increase adverse selection costs for liquidity providers in BRK.A. At the same time, our analysis highlights the challenge of reconciling traditional liquidity metrics with the current system of trade reporting for fractional trades. Most notably, the vast majority of non-exchange trades in BRK.A did not reflect the trade and settlement of whole shares and thus did not reflect anything close to the dollar-value of spreads actually earned for providing liquidity. Perhaps more importantly, for liquidity takers, the sudden drop in illiquidity shown in Figure 12 did not reflect an actual increase in BRK.A's liquidity but was instead created by the inflated trading volume created by FINRA's rounding-up rule for fractional trades. In this regard, the emergence of fractional trading may result in a severe underestimation of the true cost for placing whole share trades.

B. Spreads

In this subsection, we explore how quoted and effective spreads in BRK.A and BKR.B changed surrounding the emergence of fractional trade reporting in 2021. Even if fractional trade reporting did not result in greater price impact for trades in BRK.A, the sudden, unexplained

increase BRK.A trading volume may have nevertheless caused some traders to step back from providing liquidity for either BRK.A or BRK.B. This possibility seems especially relevant during early 2021 when there appears to have been widespread uncertainty regarding the reason for the sudden increase in BRK.A trading.

Using quote data from TAQ, we first examine whether quoted spreads for BRK.A or BRK.B widened after February 16, 2021. To construct a daily measure of quoted spreads, we take the average of the quoted spread observed at the beginning of each minute of a trading day. Quoted spreads are typically wider at the beginning and end of the trading day; therefore, we limit our observations to the 330 minutes between 10:00 AM and 3:30 PM for all full trading days between January 2, 2020 and March 31, 2022.[38] To account for fluctuations in the stock prices of BRK.A and BRK.B during this time period, we scale by the quote midpoint at the beginning of the minute.

Using this measure, panels A and B of Figure 13 plot the evolution of the quoted spreads for BRK.A and BRK.B, respectively, during this time period. To facilitate comparison with reported trading volume, each panel also plots the aggregate daily dollar volume of trading due to fractional trades at Robinhood and Drivewealth (red dots) as well as the daily dollar volume of trading reported by exchanges for each security (green dots). Because fractional trading interest only appears in trades reported to FINRA, we use exchange trading as a proxy for actual, whole-share trading interest in BRK.A and BRK.B. As before, the two vertical lines reflect February 16, 2021 and October 6, 2021.

[Figure 13 about here]

Figure 13 reveals a notable increase in quoted spreads for both securities in the days immediately following February 16, 2021, the date on which a material number of Robinhood trades began appearing in the consolidated tape. In the case of BRK.A, for instance, daily spreads averaged roughly 7.8 bps between July 1, 2020 and February 12, 2021, but increased sharply during the week of February 22, 2021 when the reported dollar volume of Robinhood trades crossed $500 million. By March 10, average quoted spreads for BRK.A had increased to nearly 20 basis points, just as the reported dollar volume for Robinhood trades exceeded $1 billion. As Panel A indicates, this spike in quoted spreads was roughly on par with the spike in

[38] We exclude trading days where the market closes before 4:00 PM.

quoted spreads observed during the early days of the COVID-19 pandemic. More generally, between February 16, 2021 through June 30, 2021, quoted spreads for BRK.A averaged 9.6 basis points per day, roughly 23% higher than during the period July 1, 2020 and February 12, 2021 (*p<.0001*).

Quoted spreads for BRK.B displayed a similar increase in the initial days after February 16, 2021, but they remained elevated for a shorter duration than was the case with BRK.A. In particular, quoted spreads for BRK.B increased from an average of just 2.4 basis points between July 1, 2021 and February 12, 2021 to approximately 3.0 basis points between February 22, 2021 and March 10, 2021. Thereafter, they declined to prior levels; indeed, between February 16, 2021 and June 30, 2021, quoted spreads for BRK.B averaged 2.0 basis points, which was slightly below the average of 2.4 basis points between July 1, 2020 and February 12, 2021 (*p<.0001*).

Not surprisingly, Figure 13 also highlights a clear correlation between fractional trading and exchange trading. This is especially true with respect to BRK.B, as one might expect given that BRK.B was created to provide a more affordable means for retail traders to invest in Berkshire Hathaway. A bivariate regression of the natural log of the daily dollar volume of fractional trades in BRK.A on the natural log of the dollar value of BRK.B daily exchange trading volume yields a tightly estimated elasticity of 0.71 (robust std. error=.12). In contrast, substituting the natural log of the dollar value of BRK.A daily exchange trading volume as the regressor yields a lower elasticity of 0.42 (robust std. error=.04).

The association between exchange trading and fractional trading naturally complicates the ability to assess how the surge in volume due to FINRA's rounding-up rule was perceived by market participants. For instance, the notable increase in quoted spreads in BRK.A and BRK.B in early March 2021, as well as in BRK.A in early May 2021, were each accompanied by elevated levels of both fractional trading volume and exchange trading volume. A Factiva search for news concerning Berkshire Hathaway during the sample period further reveals that the March episode occurred shortly after the release of the company's fourth quarter results on March 1, and the May episode occurred following the release of the company's first quarter results for 2021 on April 30. In each quarter, the company's operating profits exceeded expectations, and the price of both securities traded higher over the ensuing days. Nevertheless, while most news stories during these two time periods focused on the company's operating performance and share price appreciation, others speculated about the unusually high trading volume in BRK.A. For

instance, on both March 8 and March 10 *Barron's* published two stories about the large trading volume during this time in BRK.A and cited speculation that an institutional investor was accumulating a position in the security.[39] Likewise, on May 11, *Barron's* published a story again commenting on the elevated trading volume in BRK.A, noting that a "potential large mystery buyer" may soon be revealed in an institutional investor's first quarter Form 13-F.[40]

These examples illustrate the need to distinguish between that portion of the variation in quoted spreads that is associated with the true economic trading volume in BRK.A and BRK.B from that which is associated with the inflated volume caused by Robinhood's reporting of fractional trades. We therefore decompose these two possible influences on the quoted spreads of BRK.A and BRK.B by means of estimating the following regression model of quoted spreads for each security:

$$Spread_t = \alpha + \beta_1 Midpoint_t + \beta_2 ExVolumeA_t + \beta_3 ExVolumeB_t + \beta_4 Frac_t + \beta_5 Post_t + \beta_6 Post_t x Frac_t + \varepsilon_t \quad (6)$$

where, for each day *t*, *Spread* is the natural log of the mean dollar spread across the 330 minute observations, *Midpoint* is the natural log of the mean quoted midpoint, *ExVolumeA* and *ExVolumeB* are the natural logs of the daily dollar volume of exchange trading for BRK.A and BRK.B, respectively, *Frac* is the natural log of the daily dollar volume of Robinhood and Drivewealth trades reported to the tape, and *Post* is an indicator variable for days after February 16, 2022. We run the regression separately for BRK.A and BRK.B using daily trading data during our sample period; however, we drop data for April 7, 2021 because inspection of the TAQ trading data indicate that Robinhood failed to report fractional trades to the tape on this day.

We present the results in Table 4. Our primary coefficient of interest is the interaction term for *Post* and *Frac.* Under the Frisch-Waugh-Lovell theorem, the coefficient on this term is equivalent to the estimated effect of reported fractional trades for Robinhood and Drivewealth on

[39] Andrew Bary, Berkshire Hathaway Class A Shares Have Become More Actively Traded. Why That's Important, *Barron's,* March 8, 2021 ("Berkshire Hathaway 's class A shares have had unusually high trading activity of late. And that could mean that an investor is accumulating the high-vote stock, whose dominant holder is CEO Warren Buffett."); Andrew Bary, Shares of Warren Buffett's Berkshire Hathaway Hit a Record High, *Barron's*, March 10, 2021 ("Berkshire Hathaway stock hit a record high on Wednesday, adding to recent gains amid speculation that an institutional investor is scooping up the company's super voting class A shares.")

[40] Andrew Bary, Berkshire Hathaway's Mystery Investor Could Soon Be Revealed, *Barron's,* May 11, 2021.

quoted spreads after orthogonalizing both the variation in quoted spreads and the variation in Robinhood and Drivewealth trades to the level of whole share trading (captured by *ExVolumeA* and *ExVolumeB*). Thus, the positive, significant coefficient for the interaction term in both columns indicates a positive association between fractional trade reporting and quoted spreads for BRK.A and BRK.B even after controlling for the influence of whole share trading interest on both the level of fractional trades and quoted spreads. In particular, the estimates from Table 4 indicate that a 10% increase in reported Robinhood and Drivewealth trades is associated with roughly a 1% increase in quoted spreads in BRK.A's quoted spreads. For BRK.B, Table 4 indicates that a 10% increase in reported RH and DW trades is associated with roughly a 2.7% increase in BRK.B's quoted spreads.

[Table 4 about here]

We conduct a similar analysis for effective spreads. However, when examining the association between effective spreads and fractional share reporting, we are mindful of the mechanical relationship between fractional trades and effective spreads noted in the prior subsection. In particular, the large number of Robinhood trades—because they are almost exclusively filled at the NBBO midpoint—will decrease average effective spreads, while Drivewealth trades—because they are almost exclusively filled at either the NBB or NBO—will increase average effective spreads. Accordingly, we focus only on exchange trades to avoid the bias that might arise from the trade reporting rule we investigate. Additionally, because extremely large size trades are less likely to receive significant price improvement, we exclude trades that involve 3 or more shares for BRK.A or 300 or more shares for BRK.B, which effectively excludes the largest 1% of trades occurring on exchanges for these two securities.

Similar to our examination of quoted spreads, we find evidence consistent with effective spreads widening when Robinhood began publishing its fractional trades to the tape. In Figure 14, we first present the time series variation in exchange trade effective spreads for both BRK.A and BRK.B during our sample period. The data and analysis are identical to that used for Figure 13, with the exception that our outcome of interest is the daily average of effective spreads for each security for all exchange trades occurring between 10:30 and 15:30 that meet the size criteria noted previously. Panel A (BRK.A) and Panel B (BRK.B) each reveal similar changes in effective spreads as in Figure 13. Specifically, effective spreads increased significantly for both BRK.A and BRK.B in early March 2021, as well as in BRK.A in early May 2021. They also

generally increased with reported fractional trades after October 7, 2021. Indeed, Panel B indicates an especially close correlation between fractional trading in BRK.A and effective spreads for BRK.B commencing in mid-October.

[Figure 14 about here]

As with our prior analysis, it is likely that both effective spreads and fractional trading are jointly influenced by the underlying trading interest in Berkshire Hathaway's common equity. Therefore, we again use a modified version of Equation (6) to estimate the association between average daily effective spreads for exchange trades and reported fractional trades for Robinhood and Drivewealth, after controlling for the influence of whole share trading interest on both the level of fractional trades and effective spreads. The specification we use is identical to that used for Table 4 with the exception that the outcome of interest is the natural log of average daily effective spreads for exchange trades, calculated as previously described.

We present the results in Table 5. As with quoted spreads, Table 5 indicates a statistically significant, positive association between average daily effective spreads and the interaction of *Post* and *Frac* for both BRK.A and BRK.B. For BRK.A, Table 5 indicates that a 10% increase in reported Robinhood and Drivewealth trades is associated with 1.7% expected increase in effective spreads. For BRK.B, the estimated association is nearly double the size; a 10% increase in reported Robinhood and Drivewealth trades is associated with roughly a 3.0% increase in BRK.B's effective spreads.

[Table 5 about here]

Thus, in contrast to our findings regarding price impact, our findings regarding quoted and effective spreads are more consistent with the possibility that the unexplained surge in BRK.A trading volume during 2021 may have caused at least some market participants to become less aggressive in providing liquidity in both BRK.A and BRK.B.

C. Quoted Depth

So far, our analysis of price impact indicates that the adverse selection costs for individual trades was generally the same before and after the commencement of fractional share reporting by Robinhood and Drivewealth. Yet our analysis of quoted and effective spreads also suggests the surge in BRK.A trading volume in 2021 may have induced some market participants to become less aggressive in providing liquidity. Taken together, these findings suggest

heterogeneous interpretations of BRK.A's trading volume in 2021, perhaps with specialists and market-makers understanding it for what it was, while a segment of other liquidity providers viewed it with more uncertainty.

To assess this possibility in more detail, we explore in this subsection quoted depth in BKR.A during 2020 and 2021, but we do so in a particular fashion. Specifically, if fractional share reporting produced these heterogeneous effects among traders, we would expect them to be manifested differentially in the willingness of market participants to offer to sell shares of BRK.A at different prices. That is, liquidity providers and specialists (who we refer to as "*active price-setters*") making a market at the Offer should be expected to quote at the top of the order book in largely the same quantities before and after February 16, 2021 (though perhaps at less aggressive prices). In contrast, holders of BRK.A who typically post offers deeper in the order book (who we refer to as "*passive price-setters*") may have chosen to step to the sidelines given the possibility that a large investor was acquiring a position in BRK.A.

Because this analysis requires the full order book for BRK.A, we turn to data from MayStreet, which has the unique advantage of providing minute-by-minute data for the entire order book for both securities. As with our analysis of spreads, we limit our analysis of BRK.A's order book to the 330 minutes between 10:00 AM and 3:30 PM for all full trading days between January 2, 2020 and March 31, 2022. For each minute, we calculate the midpoint of the best bid and offer, and we divide posted offers for the minute into two categories based on their distance of the offer price from the midpoint. Specifically, if an order is priced within 5 basis points of the quote midpoint, we assign the order to the *active price-setters* category; otherwise, an order is assigned to the *passive price-setters* category. Lastly, we aggregate the total dollar value of offers at the start of each minute for each category.

In Figure 15, we plot the natural log of the daily average for the two categories of offers during our sample period. Panel A presents the results for orders assigned to the *active price-setters* category. As shown in the panel, the data reflect a stark drop in the dollar depth on offer during March 2020 during the onset of COVID-19 pandemic in March 2020. Panel A also reveals a somewhat less stark decline in dollar depth in early March 2021. This latter decline coincided with the surge in Robinhood's fractional trades, but as noted previously, it also followed the release of Berkshire's financial results for the first quarter of 2021. The conflation of these two events thus complicates identifying the primary reason for the drop in dollar depth

on offer in early March 2021. That it subsequently rebounded in late spring 2021, however, suggests that whatever effect the sudden surge in BRK.A's volume may have had on the most aggressive liquidity providers, it was short-lived.

[Figure 15 about here]

In contrast, Panel B suggests a substantially different story for passive price-setters. Following a discontinuous increase in these less aggressive orders on February 28, 2020, the dollar value of orders at these less aggressive prices remained generally stable through the extreme volatility of March 2020 and thereafter increased gradually until mid-February 2021, reaching a peak of approximately $206 million on February 12, 2021. On February 16, 2021, it commenced a rapid and sharp decline to roughly $50 million where, except for a brief rebound to approximately $80 million during April 2021, it generally remained for the duration of the sample period.

Overall, these findings are thus consistent with the commencement of fractional share reporting having heterogeneous effects on the market makers who set the NBBO and other liquidity providers who supply liquidity as less aggressive prices.

5. What to do about fractional shares?

The current regulatory reporting protocols for fractional share trades seem far from optimal. Two problems are immediately apparent. As we have demonstrated, the current FINRA rule requiring immediate reporting of rounded-up trades to the tape distorts reported trade volumes, obscures determining the actual amount of fractional trading, and in our view interferes with the efficient functioning of the market. A second problem is the lack of uniformity in reporting protocols; depending upon the underlying broker handing the trade, a fractional share trade may be executed internally, and so handled very differently in the reporting montage. Either approach seems to miss the mark of enhancing the transparency and operation of the market.

The basic problem is that fractional shares don't really fit into our current equity market structure. No exchange will accept a fractional order and so fractional executions have to occur elsewhere. In this regard, the issues here are remarkably similar to the problems that long plagued odd-lot trading. Odd-lots (or orders for less than 100 shares) were traditionally handled by brokerage firms who filled these small orders out of long and short positions acquired through round lot trading, much as Apex Clearing operates today. The SEC Special Study on Market

Structure (1963) noted that 99% of odd lot trades were handled by just two brokerage firms. Like fractional shares, these odd-lot trades were largely invisible; they were not included in the "ticker" that showed stock trades or in its successor, the consolidated tape. They were also expensive. Odd lot trades were charged an "odd lot differential" price of 1/4 from the quotes (recall that stocks traded in eights at the time).

The NYSE instituted a specific trading system for odd lots in 1976 to provide improved, but separate, executions for these small trades, with this system only decommissioned in 2010. Odd lot orders are now allowed on all exchanges and beginning in 2014 odd lot trades are included in the consolidated tape (see O'Hara, Yao, and Ye (2014) for analysis of these hidden odd lot trades). Despite this greater transparency, problems remain with respect to odd lot execution costs (see Bartlett (2021)) and transparency and liquidity issues (see Bartlett, McCrary, and O'Hara (2022a)).

Is there a better way to handle fractional share trading? Certainly, having transparency in trade reporting is a laudable goal, but as this paper suggests it is a means to an end (i.e. better markets) and not an end in itself. Reporting (and printing) each actual fractional share amount to the consoldiated tape would be completely transparent and is certainly feasible. After all, Robinhood and Drivewealth are already reporting their fractional trader to the tape (which, across all securities, amount to roughly 2-5 million trades per week based on the OTC Transparency data). The problem is they are inaccurately reporting each trade as a trade for a whole share. However, simply requiring the reporting of actual fractional trade sizes does not solve the problem of the two disparate reporing systems currently in use. And perhaps more important, as fractional trading continues to grow in popularity, this may add unnecessary noise to the tape without adding much in the way of corresponding benefits. Not reporting at all, however, seems worse—fractional shares are fast becoming important in retail trading and ignoring such trades is overly restrictive.

There seem to be two possible middle-ground solutions. Perhaps the easiest is to simply flag the current reported fractional trades with an asterisk denoting that this trade is for an amount less than one share. This expedient solution provides information to the market without disrupting the current reporting protocal. But it does not solve the problem of disparate reporting protocols. A second potential solution is to require reporting of aggregated trades that total one share. Thus, Robinhood, for example, would execute fractional shares in a stock internally and

report to the tape when these trades aggregate to one share, much as currently happens with Apex Clearing. A variant of this approach is to require real time reporting of trades with their actual amounts to the SIPs (Security Information Processors) but the SIPs then only print to the consolidated tape the aggregated trade at its VWAP. This has the advantage of making fractional share reporting essntially consistent across the two approaches currently used. There are surely many other possible approaches to this problem, but as the SEC contemplates changes to the National Market System consolicated equity market data plan, we suggest that the issues connected with fractional share reporting be part of any new framework.[41]

6. Conclusion

Fractional share trading may revolutionize retail trading but perhaps not without growing pains. This paper has demonstrated how current reporting protocols surrounding fractional share trading have massively inflating the reported volume of BRK.A, causing dislocations in its relationship to its paired stock BRK:B, missed arbitrage opportunities for a confused market, and impacts on liquidity and spreads. We argue that a well-intentioned but ultimately distortionay FINRA rule can explain the mysterious increase in Berkshire-Hathway A share volume that occassioned all these effects.

Our research also suggests some broader conclusions. Certainly one is how important it is to get the microstructure of markets right. Fractional shares do not currently fit into our current market structure, and partial attempts to make them do so can miss the mark. When this compromises the information structure of the market, the effects can be wide-ranging. What may be surprising to some is how a few misreported shares can have such large effects in a supposedly efficient market. But efficiency is a fragile concept, resting as it does on the information available to support it. It is a testament to how much the market trusts our current system that the misunderstood reporting of fractional shares engendered such pervasive market effects, at least among some market participants. Going forward, the market will learn but as our paper has established this learning can take some time.

[41] There are various proposals for changes to the consolidated tape system including competing SIPs and additions to the data montage. For more information, see https://www.sec.gov/rules/sro/nms.htm#4-757

Fractional share trades, much like odd lot trades before them, will likely continue to grow in importance. How to incorporate them into the market structure may be the real stock market mystery to solve.

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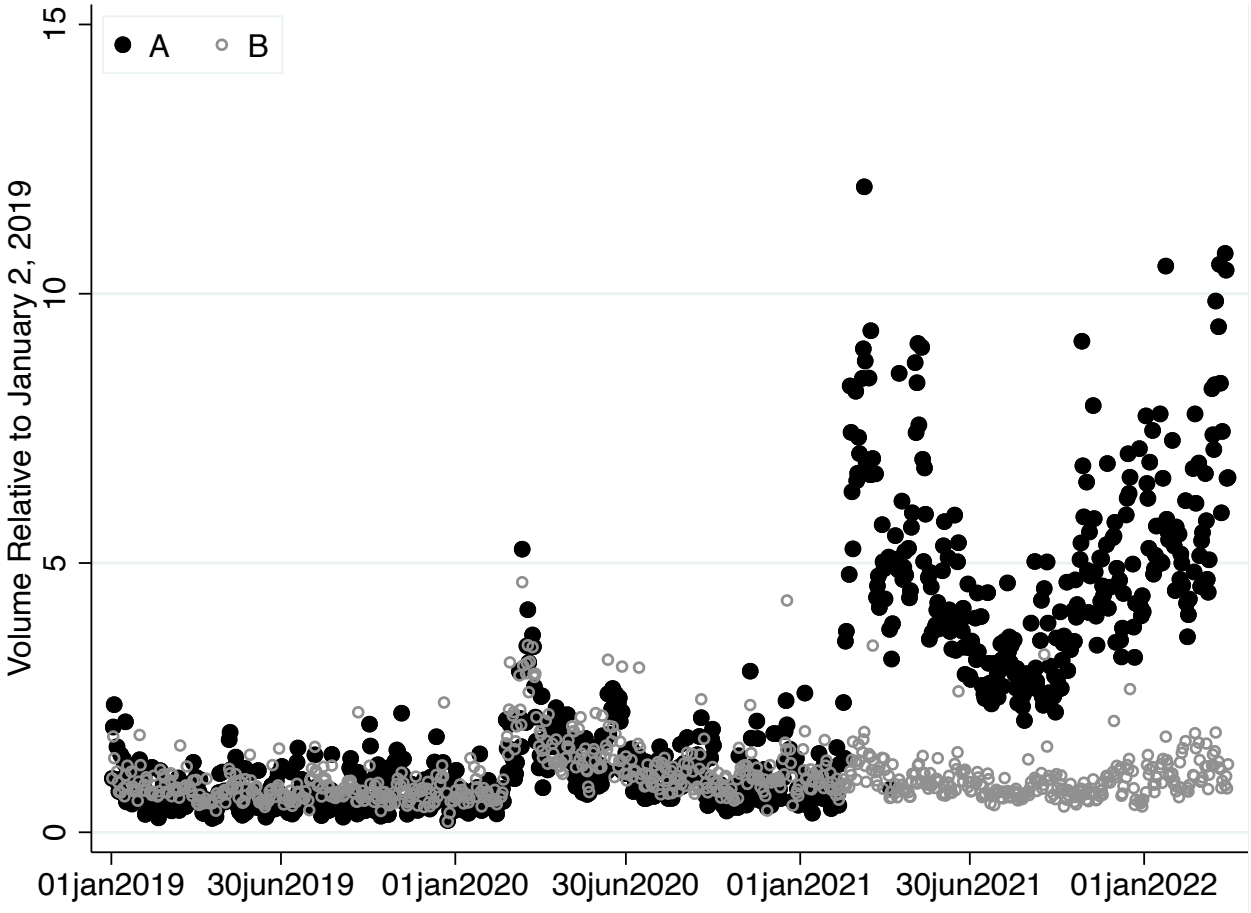
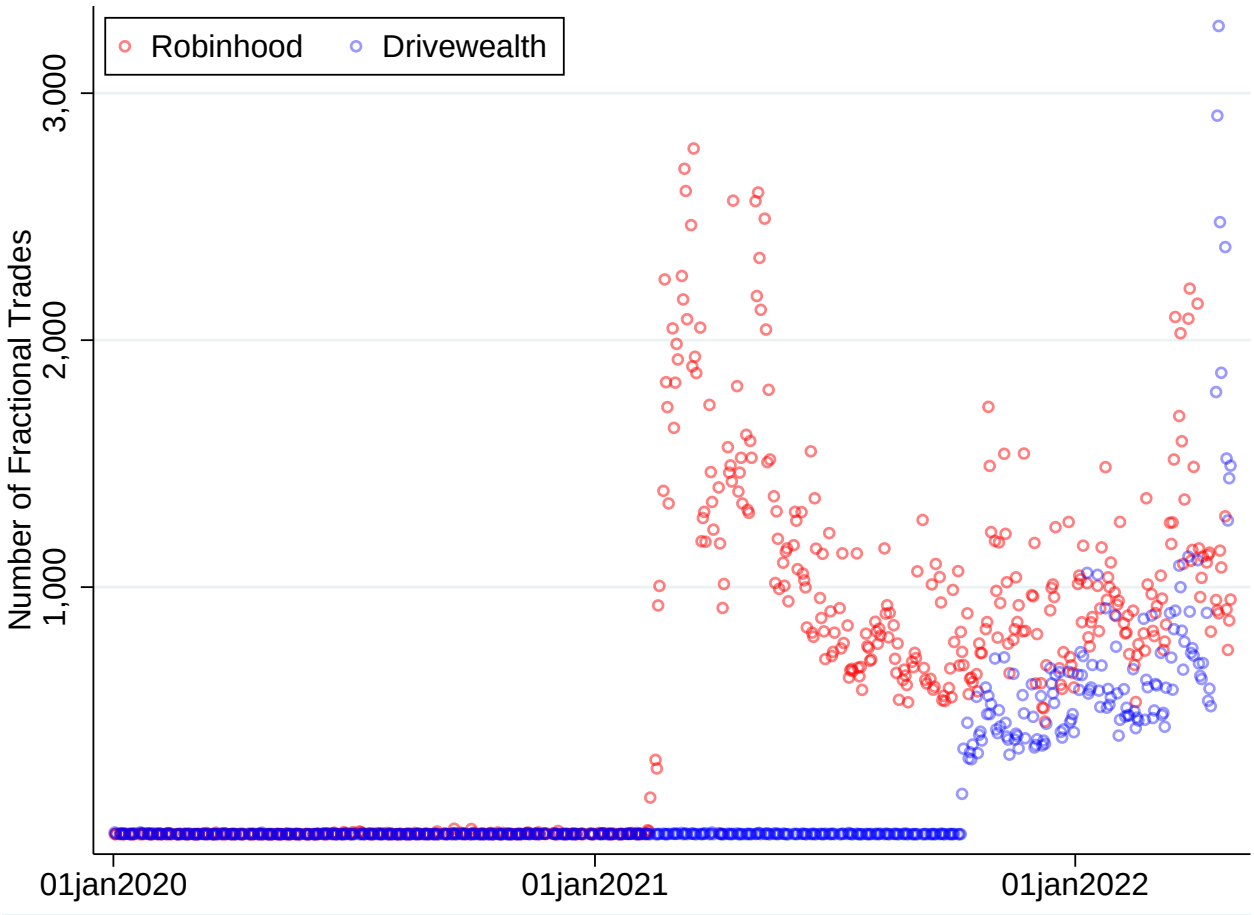
Figure 1. Volume of Trading in BRK.A vs. BRK.B, 2019-2022



Note: Figure presents the daily number of shares traded for BRK.A (solid circles) and BRK.B (hollow circles) relative to the daily number of shares traded for each security on January 2, 2019.

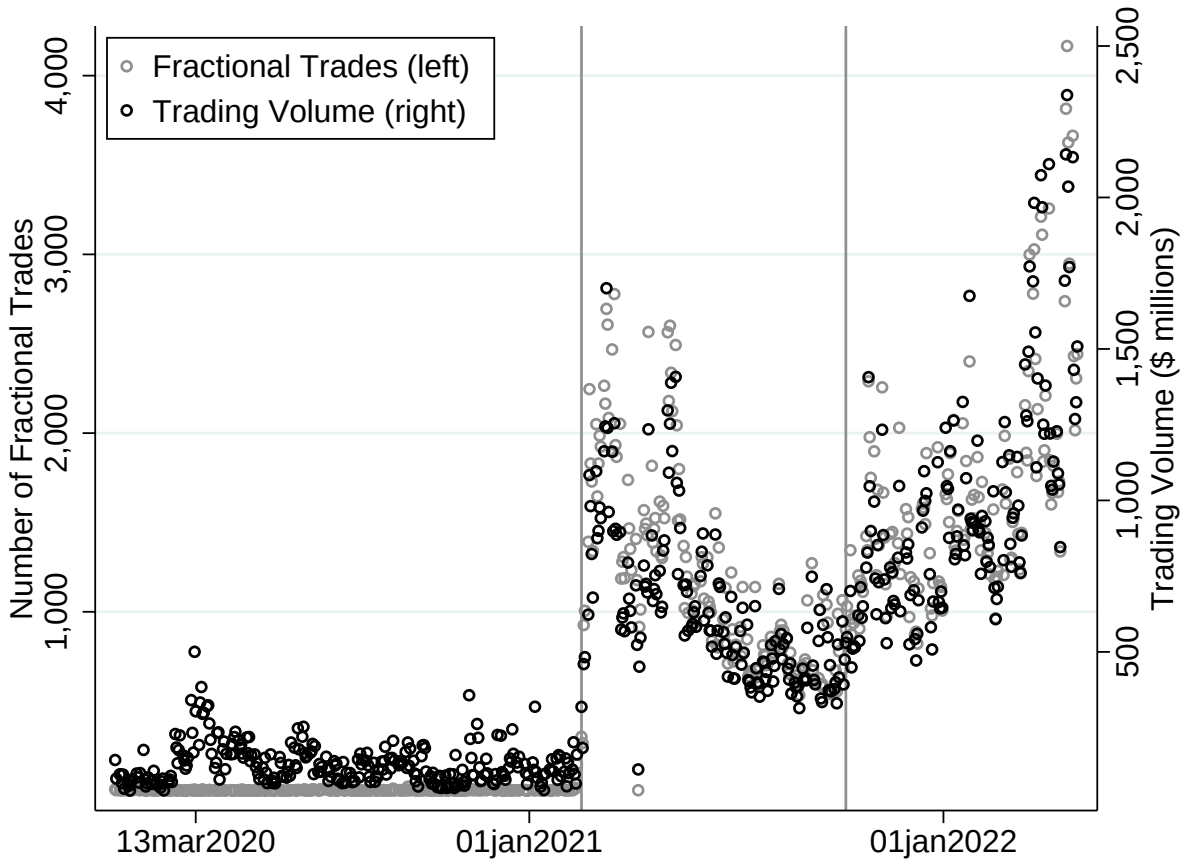
Figure 2. Estimated Fractional Trades in BRK.A:

Robinhood and Drivewealth Proxies



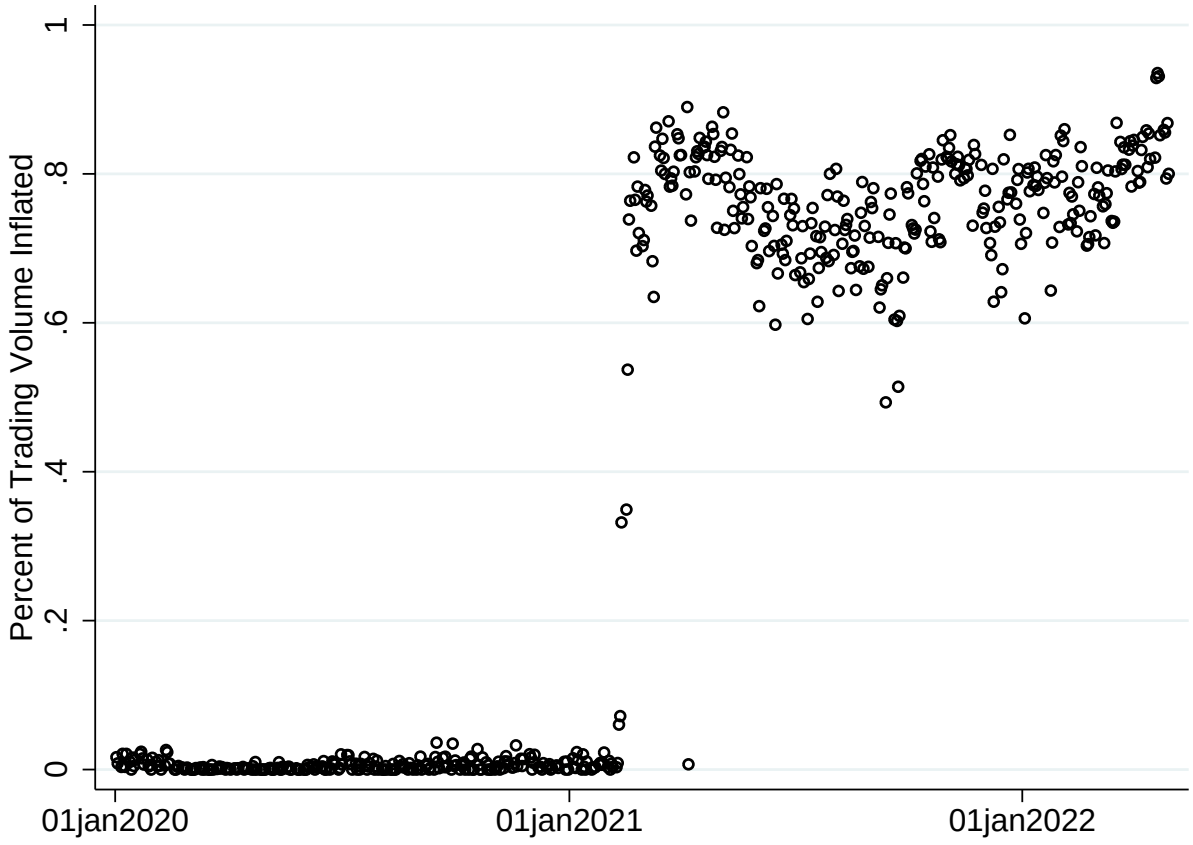
Note: Robinhood and Drivewealth proxies for fractional trades based on fields in TAQ data. See text for details.

Figure 3. BRK.A Fractional Trades and Total Trading Volume



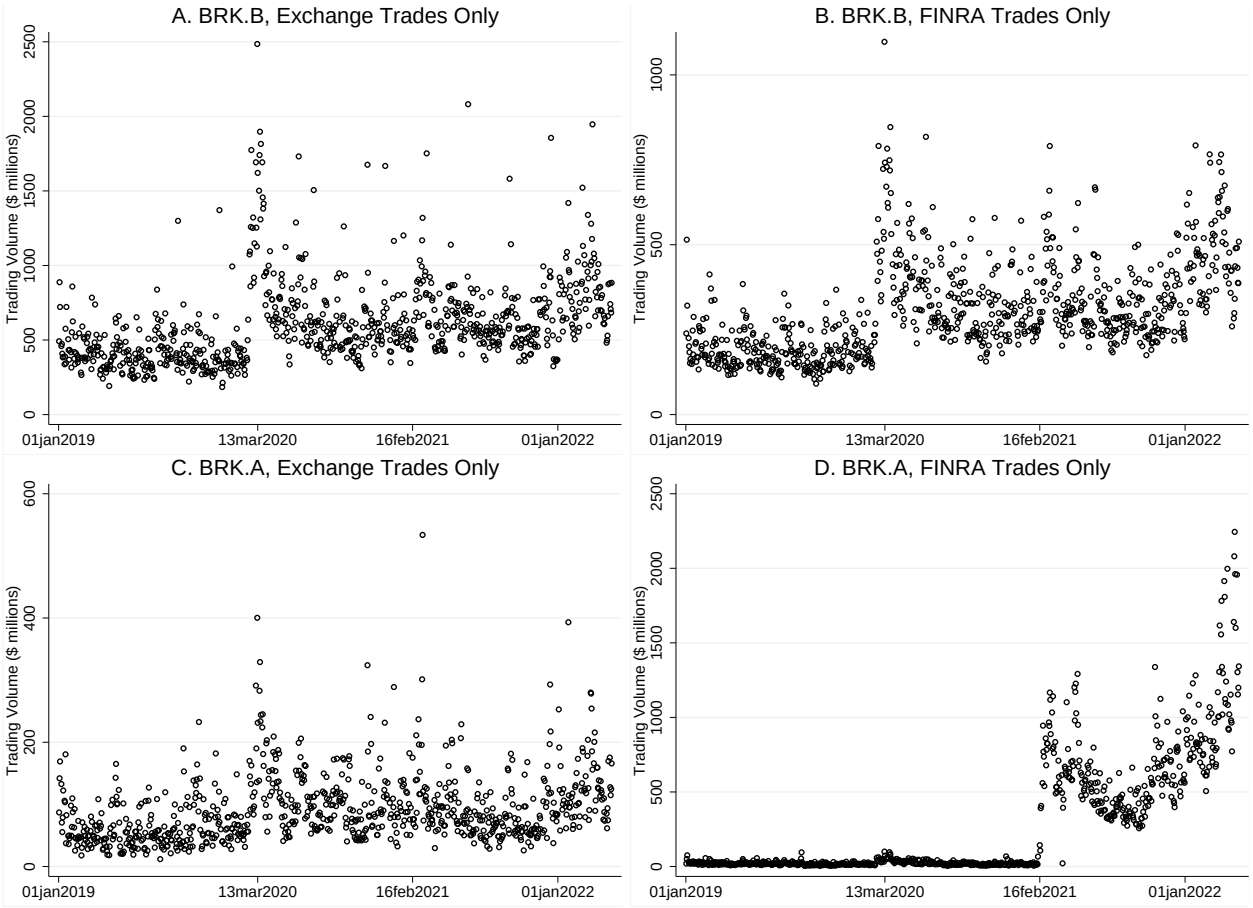
Note: Fractional trades estimated as the sum of the Robinhood and Drivewealth proxies. See notes to Figure 2 and text for details.

Figure 4. Fraction of Trading Volume in BRK.A Inflated by the "Round Up" Rule



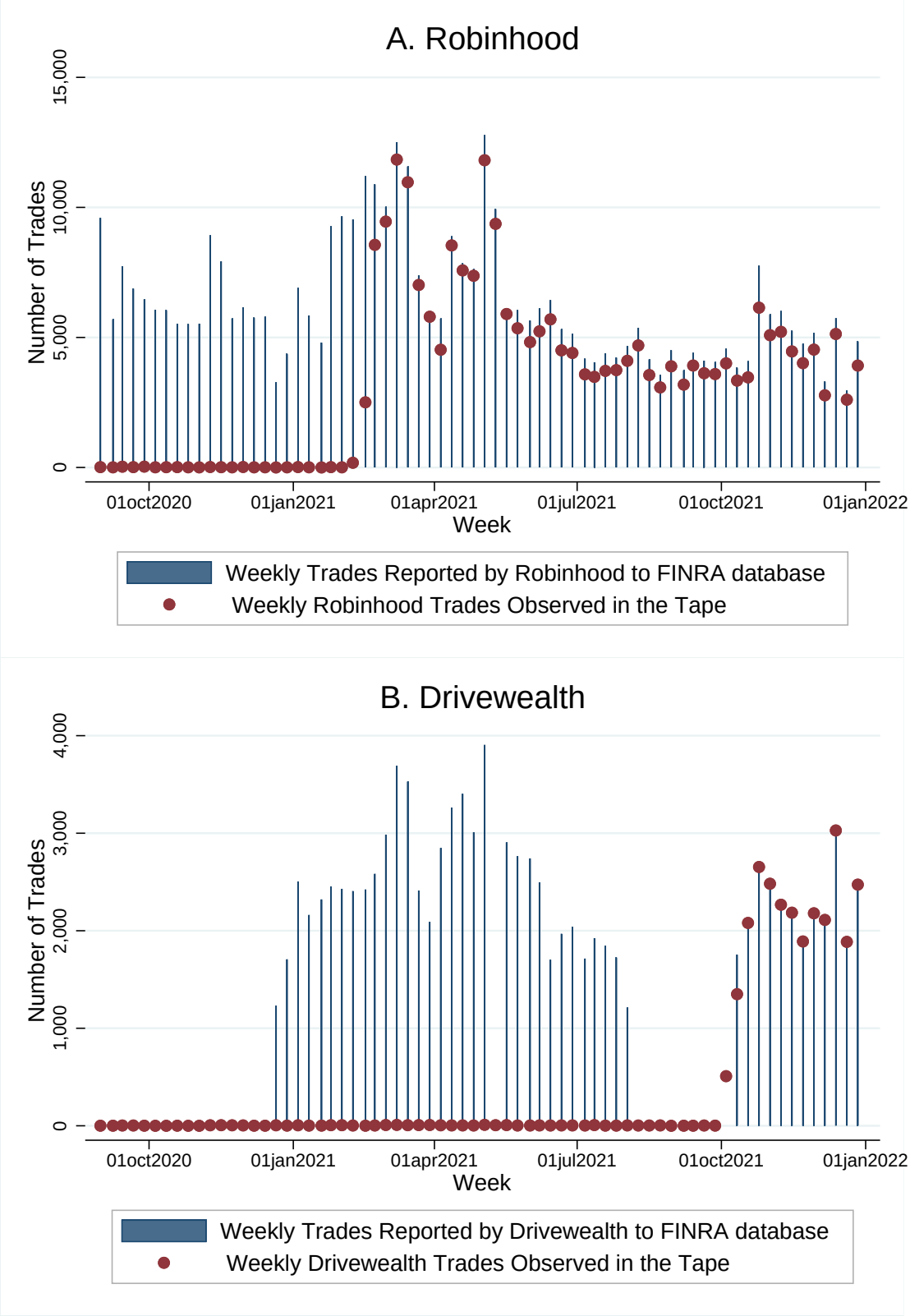
Note: Estimate of inflation assumes each Robinhood and Drivewealth trade is backed by $240, the median account size of Robinhood accountholders in 2021. See text for details.

Figure 5. Dollar Volume of Trading, FINRA and Exchange Trades: BRK.B, BRK.A



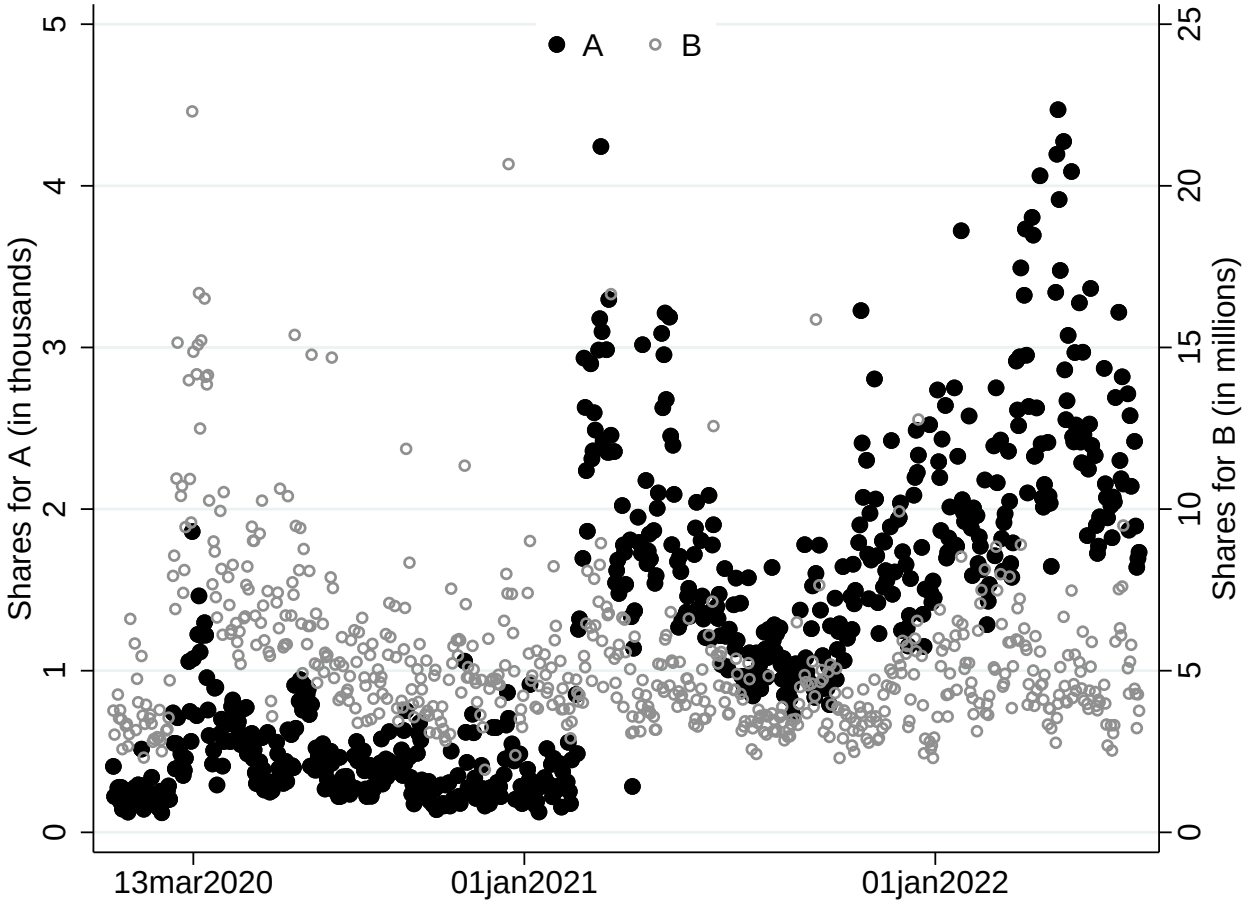
Note: See notes to Figure 1.

Figure 6. Reporting to FINRA Database vs. TAQ Proxy: BRK.A



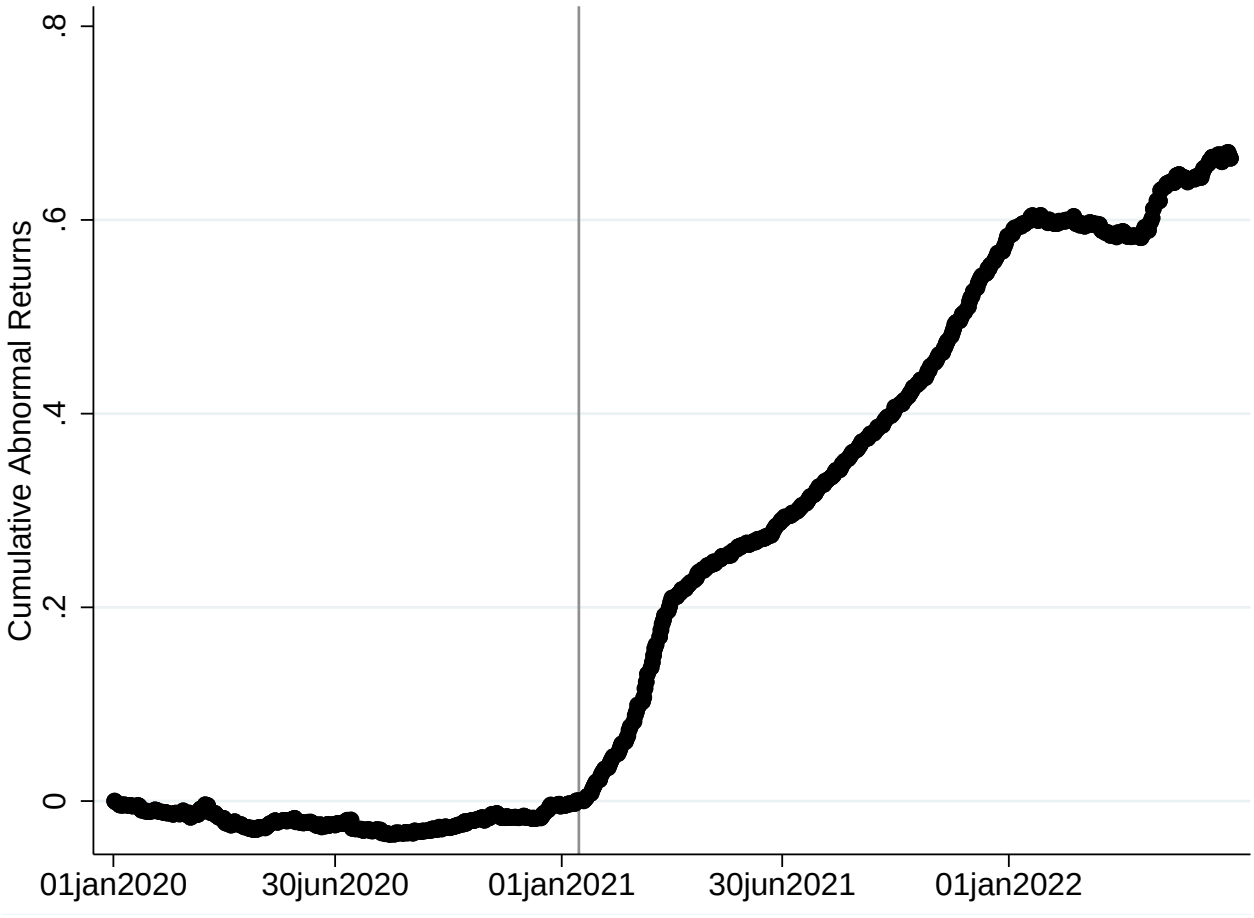
Note: FINRA database is also known as the OTC Transparency data. Both Robinhood and Drivewealth backfilled the FINRA database (and Drivewealth is ongoingly), but there is no backfilling of the public tape. See text for details.

Figure 7. Shares Traded in BRK.A and BRK.B



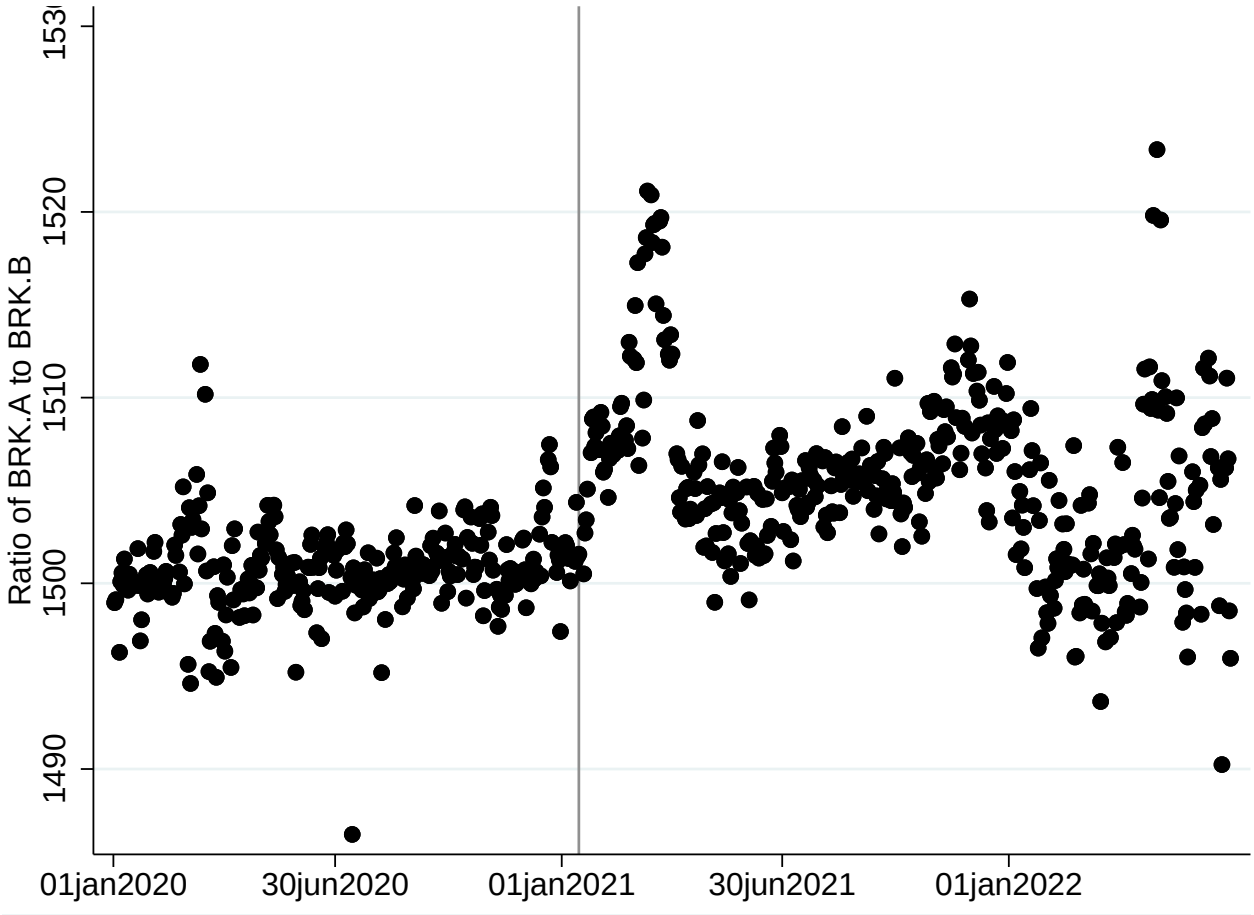
Note: Total shares traded are from CRSP until 3/31/2022 and are predicted CRSP total shares based on intraday TAQ total shares traded thereafter.

Figure 8. Cumulative Abnormal Returns to BRK.A



Note: See text for details.

Figure 9. The Ratio of BRK.A to BRK.B Over Time



Note: See text for details.

Figure 10. Effective Spreads, Exchange Trading Volume, and Trading Volume Reported by Robinhood and Drivewealth for BRK.A, January 2020-March 2021

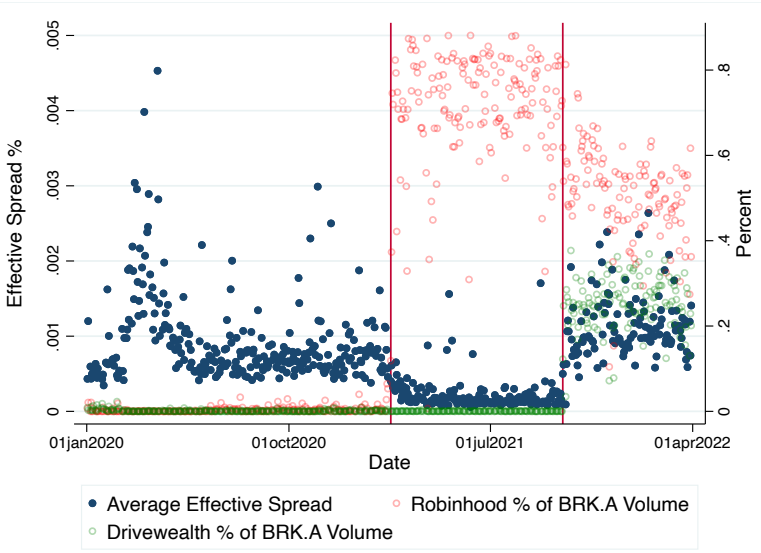


Figure 11. Price Impact Estimates for BRK.A, January 2020-March 2021

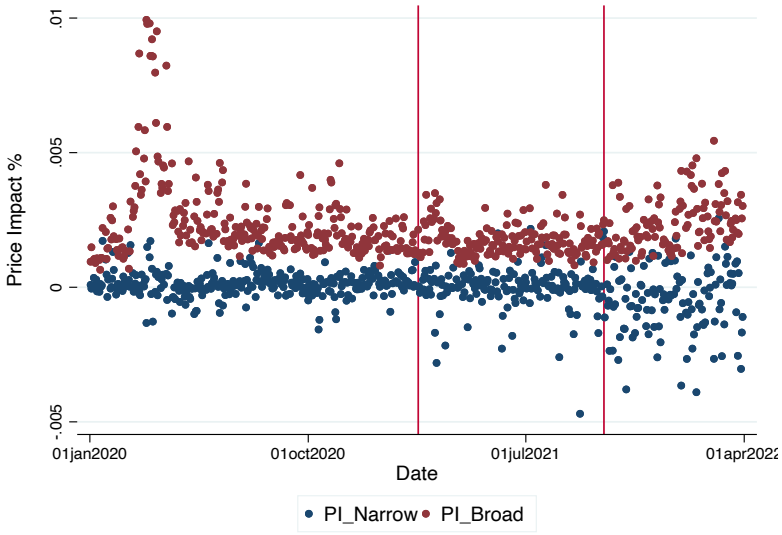


Figure 12. Amihud Illiquidity for BRK.A, January 2020-March 2021.

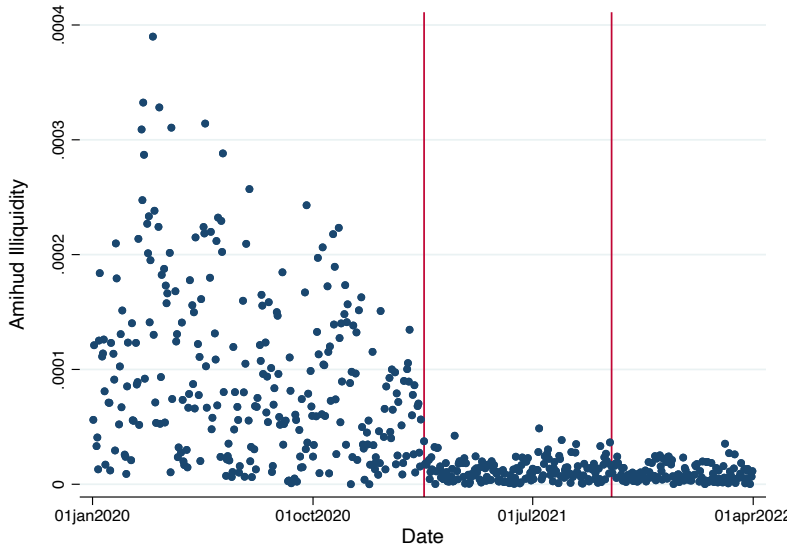
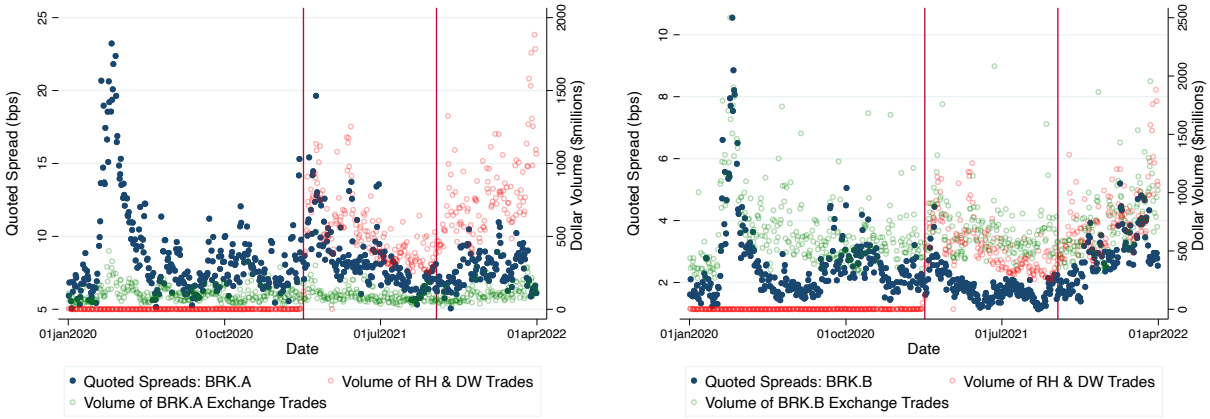
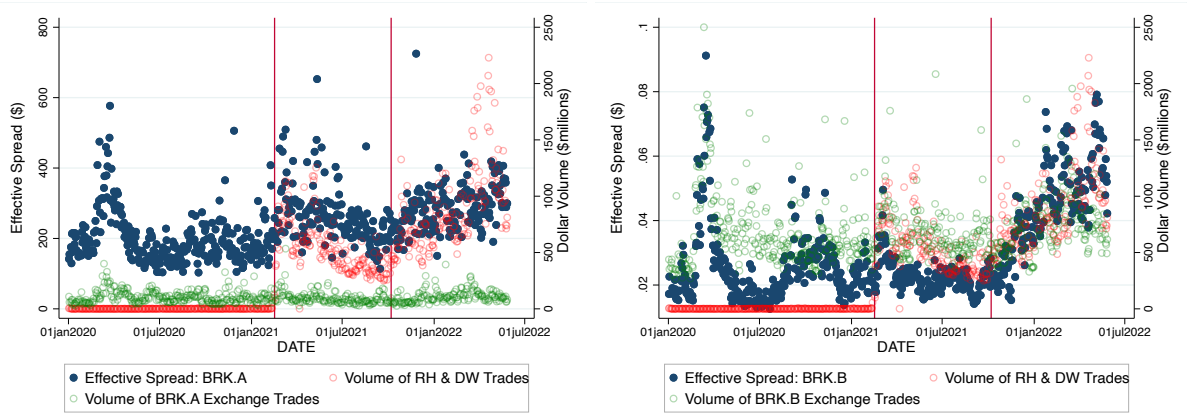


Figure 13. Quoted Spreads and Trading Volume BRK.A vs. BRK.B



A B

Figure 14. Effective Spreads and Trading Volume BRK.A vs. BRK.B



A B

Figure 15. Aggregate Dollar Value of Posted Offers in BRK.A by Active Price-Setters vs. Passive Price-Setters, January 2020-March 2022.

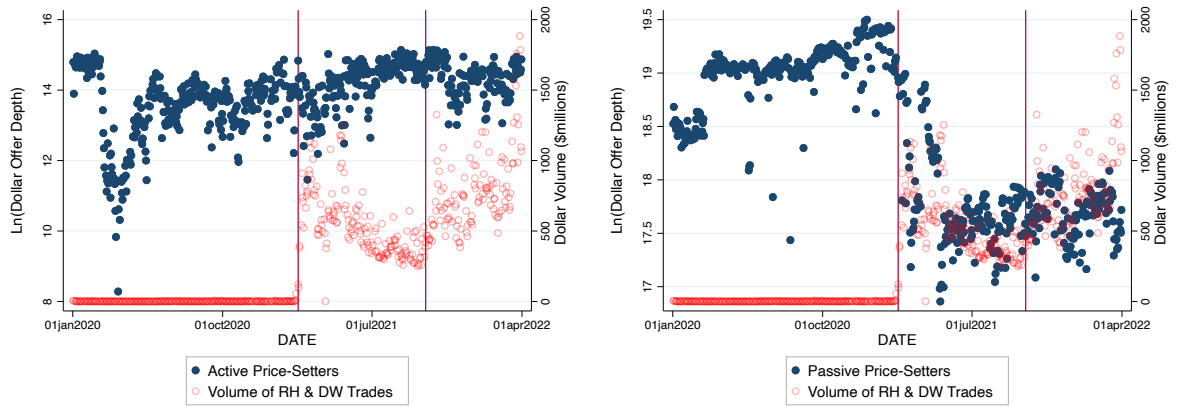


Table 1. Unit Root Tests for BRK.A, BRK.B, and Disequilibrium

A. Dickey-Fuller Approach

	Test Statistic		Critical Values for Test		
	Intraday VWAP	Closing Price	0.01	0.05	0.10
log BRK.A	-0.852	-1.000	-3.43	-2.86	-2.57
log BRK.B	-0.856	-1.045	-3.43	-2.86	-2.57
Fitted residuals	-10.342	-15.877	-3.43	-2.86	-2.57
B. Phillips-Perron Approach: τ					
log BRK.A	-0.946	-0.941	-3.43	-2.86	-2.57
log BRK.B	-0.949	-0.948	-3.43	-2.86	-2.57
Fitted residuals	-10.444	-17.822	-3.43	-2.86	-2.57
C. Phillips-Perron Approach: ρ					
log BRK.A	-1.779	-1.796	-20.7	-14.1	-11.3
log BRK.B	-1.812	-1.837	-20.7	-14.1	-11.3
Fitted residuals	-189.158	-506.853	-20.7	-14.1	-11.3

Table 2. Estimating the Cointegrating Relationship

	Dependent Variable: log BRK.A	
Covariate	Intraday VWAP	Closing Price
log BRK.B	1.0053	1.0038
	(0.0006)	(0.0009)
	[0.0016]	[0.0018]
Intercept	7.2864	7.2942
	(0.0031)	(0.0048)
	[0.0088]	[0.0098]

Note: Heteroskedasticity-robust standard errors in parentheses.
Newey-West standard errors (20 lags) in square brackets.
Sample size is 630 trading days.

Table 3. Models for Returns for BRK.A

A. Intraday VWAP Before Jan 15, 2021

Covariate	Model (3)	Model (4)	Model (5)	Model (6)
Returns to BRK.B	0.9735	0.9658	0.9619	0.9677
	(0.0077)	(0.0073)	(0.0071)	(0.0074)
	[0.0089]	[0.0088]	[0.0084]	[0.0106]
Disequilibrium:		-0.3143		
lagged log (BRK.A/BRK.B)		(0.0401)		
		[0.0840]		
Disequilibrium when positive			-0.1106	-0.5531
			(0.0591)	(0.1245)
			[0.0828]	[0.1694]
Disequilibrium when negative			-0.6289	-0.9856
			(0.0747)	(0.0840)
			[0.1080]	[0.0990]

B. Closing Price

Covariate	Model (3)	Model (4)	Model (5)	Model (5)
Returns to BRK.B	0.9294	0.9452	0.9450	0.9447
	(0.0353)	(0.0247)	(0.0246)	(0.0269)
	[0.0359]	[0.0237]	[0.0233]	[0.0225]
Disequilibrium:		-0.5372		
lagged log (BRK.A/BRK.B)		(0.1067)		
		[0.1737]		
Disequilibrium when positive			-0.4928	-0.9954
			(0.2238)	(0.2416)
			[0.2737]	[0.3201]
Disequilibrium when negative			-0.6042	-0.8760
			(0.1461)	(0.0165)
			[0.0968]	[0.0768]
Sample size	629	629	629	261

Note: Heteroskedasticity-robust standard errors in parentheses. Newey-West standard errors (20 lags) in square brackets. See text for details.

Table 4: Estimates of Quoted Spreads for BRK.A and BRK.B

	(1)	(2)
	BRK.A	BRK.B
Midpoint	-0.0461	1.297***
	[0.130]	[0.168]
ExVolumeA	0.0427	0.125***
	[0.0262]	[0.0368]
ExVolumeB	0.233***	0.244***
	[0.0435]	[0.0667]
Frac	-0.00606	-0.0186
	[0.0191]	[0.0196]
Post	-0.286*	-1.803***
	[0.173]	[0.253]
Post X Frac	0.0952***	0.273***
	[0.0329]	[0.0445]
Constant	4.443***	-12.09***
	[1.681]	[0.963]
Observations	482	482
R-squared	0.448	0.414

Note. Heteroskedasticity-robust standard errors in parentheses. *,**, and *** indicate significance levels of 10%, 5%, and 1%. See text for details.

Table 5: Estimates of Effective Spreads for BRK.A and BRK.B

	(1)	(2)
	BRK.A	BRK.B
Midpoint	0.0118	1.205***
	[0.161]	[0.174]
ExVolumeA	0.00996	0.130***
	[0.0305]	[0.0375]
ExVolumeB	0.252***	0.265***
	[0.0471]	[0.0671]
Frac	-0.0352	-0.0275
	[0.0237]	[0.0202]
Post	-0.561**	-1.956***
	[0.229]	[0.259]
Post X Frac	0.172***	0.303***
	[0.0432]	[0.0454]
Constant	3.406	-12.50***
	[2.073]	[1.002]
Observations	482	482
R-squared	0.386	0.403

Note. Heteroskedasticity-robust standard errors in parentheses. *,**, and *** indicate significance levels of 10%, 5%, and 1%. See text for details.